MATHEMATICAL SCIENCES INSTITUTE BELAGAVI
Newsletter of MSI-Belagavi
VOL-III, NUMBER 1

# CELEBRATING MATHEMATICAL COMMUNICATION


- ANDRE-WEIL'S LETTERS
- MERSENNE - PASCAL LETTERS

# MSI Newsletter

**Vol III - Number 1**

## Special Issue: Mathematical Communication

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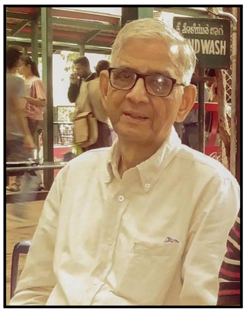
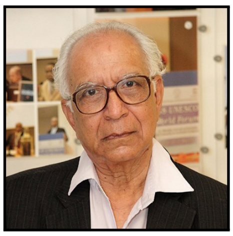


Our beloved friend and probabilist K B Athreya passed away on 24th March 2023. We have paid homage to this great academician. He was almost family to MSI Belagavi

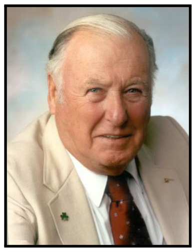
Along with K B Athreya we also pay tributes to C. R.Rao one of the legendary statistician.

K. R. Parthasarathy renowned mathematician passed away June 2023, so we have a written a tribute article on him.



We have covered a famous interview of late Prof. M. S. Narsimhan



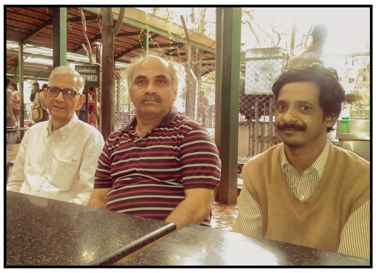


Reflections: In continuation to the applied mathematics aspects in the classics, here we reflect on the work of James-Cooley

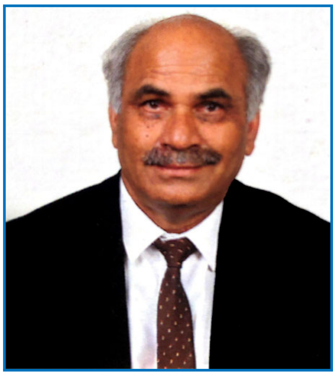
We see what is happening in mathematical sciences

We have chronicled life and works of George Polya.

Tulsi Srinivasan



# From the Desk of Director, MSI



*T. Venkatesh, Director, MSIB*

I am happy to see this issue of the Newsletter Bulletin. Last time we had announced the focal theme as computational sciences and so we can see the formalization of mathematics through the legendary minds of the the 15th century up untill 21st century. The cover page illustrates mathematical communication in those periods. We have eloborated on Fr. Mersenne and his many ways of propagating mathematical ideas. We also chronicle contemperories like Blaise Pascal, Pierre de Fermat Leibnitz and others.

Another very important historical aspect covered in this issue is the saga of David Hilbert toward formalizing mathematical proof systems. We have been highlighting our efforts towards the specialization of history of mathematics, since it is in a way driving the curiosity of young minds who want to enter the realm of mathematical research.

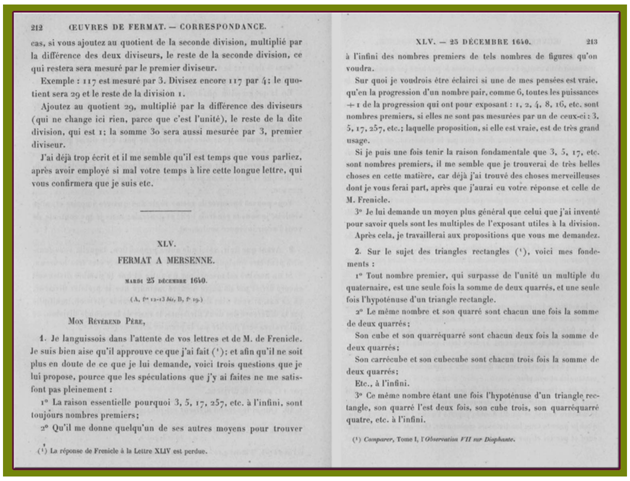
In this issue we pay our tributes to some of the departed legends in Mathematics. First we pay our homage to the Indian mathematician Professor M. S. Narasimhan. We have also covered his interview in the face2face section. In the classics section we have the classic work of George Polya. Very significantly this makes a nice read especially for young researchers because he emphasised so much on teaching and learning of Mathematics.

There is this unfortunate news.....our beloved friend Prof. Krishna B Athreya breathed his last in the month of March this year. So also we heard the passing away of the international statistics prize winner C R Rao and K. R. Parthasarathy the quantum probability expert. We pay homage to these stalwarts in the obituary section.

Our Math-education section also focusses on several techniques of problem solving and other things. Also the author writes about the Inquiry Based Learning (IBL). From this issue we are calling a section, Math-in media rather than Y-space that was covered in the last bulletin. We thought that one can review mathematics from any media, be it social media, movies or theater. Our Book review focusses on the book "The Weil Conjectures"...

We have started a new section in our bulletin on REFLECTIONS. Here we recreate some classic work of the yesteryears. This time we have chronicled the work by James William Cooley.

# Marin Mersenne and his letter correspondences.



212 ŒUVRES DE FERMAT. — CORRESPONDANCE.

cas, si vous ajoutez au quotient de la seconde division, multiplié par la différence des deux diviseurs, le reste de la seconde division, ce qui restera sera mesuré par le premier diviseur.

Exemple : 117 est mesuré par 3. Divisez encore 117 par 4; le quotient sera 29 et le reste de la division 1.

Ajoutez au quotient 29, multiplié par la différence des diviseurs (qui ne change ici rien, parce que c'est l'unité), le reste de la dite division, qui est 1; la somme 30 sera aussi mesurée par 3, premier diviseur.

J'ai déjà trop écrit et il me semble qu'il est temps que vous parliez, après avoir employé si mal votre temps à lire cette longue lettre, qui vous confirmera que je suis etc.

XLV.

FERMAT A MERSENNE.

MARDI 25 DÉCEMBRE 1640.

(A, f^os 12-13 *bis*, B, f° 19.)

Mon Révérend Père,

1. Je languissois dans l'attente de vos lettres et de M. de Frenicle. Je suis bien aise qu'il approuve ce que j'ai fait (1); et afin qu'il ne soit plus en doute de ce que je lui demande, voici trois questions que je lui propose, pource que les spéculations que j'y ai faites ne me satisfont pas pleinement :

1° La raison essentielle pourquoi 3, 5, 17, 257, etc. à l'infini, sont toujours nombres premiers;

2° Qu'il me donne quelqu'un de ses autres moyens pour trouver

(1) La réponse de Frenicle à la Lettre XLIV est perdue.

XLV. — 25 DÉCEMBRE 1640. 213

à l'infini des nombres premiers de tels nombres de figures qu'on voudra.

Sur quoi je voudrois être éclairci si une de mes pensées est vraie, qu'en la progression d'un nombre pair, comme 6, toutes les puissances +1 de la progression qui ont pour exposant : 1, 2, 4, 8, 16, etc. sont nombres premiers, si elles ne sont pas mesurées par un de ceux-ci : 3, 5, 17, 257, etc.; laquelle proposition, si elle est vraie, est de très grand usage.

Si je puis une fois tenir la raison fondamentale que 3, 5, 17, etc. sont nombres premiers, il me semble que je trouverai de très belles choses en cette matière, car déjà j'ai trouvé des choses merveilleuses dont je vous ferai part, après que j'aurai eu votre réponse et celle de M. Frenicle.

3° Je lui demande un moyen plus général que celui que j'ai inventé pour savoir quels sont les multiples de l'exposant utiles à la division.

Après cela, je travaillerai aux propositions que vous me demandez.

2. Sur le sujet des triangles rectangles (1), voici mes fondements :

1° Tout nombre premier, qui surpasse de l'unité un multiple du quaternaire, est une seule fois la somme de deux quarrés, et une seule fois l'hypoténuse d'un triangle rectangle.

2° Le même nombre et son quarré sont chacun une fois la somme de deux quarrés;

Son cube et son quarréquarré sont chacun deux fois la somme de deux quarrés;

Son carrécube et son cubecube sont chacun trois fois la somme de deux quarrés;

Etc., à l'infini.

3° Ce même nombre étant une fois l'hypoténuse d'un triangle rectangle, son quarré l'est deux fois, son cube trois, son quarréquarré quatre, etc. à l'infini.

(1) *Comparer*, Tome I, l'*Observation VII sur Diophante*.

*F*ermat Letter credits researchgate.net

*Talking about letter correspondences we have one interesting story involving Blaise Pascal, Fermat and some contemperory mathematicians.*

We are talking about **Fr Marin Mersenne** - Mersenne was a prolific correspondent, and he maintained a vast network of contacts throughout Europe. He used this network to share information and ideas, and to facilitate communication between scientists and scholars. In those days people in Europe believed that if some news has to be propagated Mersenne was the right man to do so through ot France and other places.

He would often send letters to multiple people at the same time, and he would ask them to forward the letters to other people. This allowed him to quickly and easily share information and ideas with a large number of people. Father Marin Mersenne (1588-1648), who lived in France during an era of intellectual giants, from his country and elsewhere: Rene Descartes was in a different region of France who lived during 1599-1651. Blaise Pascal (1623-1662), Pierre de Fermat (1601-1665), were also his contemperories as we can see.

Piere de Fermat was a lawyer by profession but doing mathematics was his hobby. **Blaise Pascal** was a very influential French mathematician and philosopher who contributed to many areas of mathematics. He worked on conic sections and projective geometry and in correspondence with Fermat he laid the foundations for the theory of probability. He was the third of Etienne Pascal's children. According to his fathers; orders Blaise was not to study mathematics before the age of 15 and all mathematics texts were removed from their house. Blaise however, his curiosity raised by this, started to work on geometry himself at the age of 12. He discovered that the sum of the angles of a triangle are two right angles and, when his father found out, he relented and allowed Blaise a copy of Euclid. At the age of 14 Blaise Pascal started to accompany his father to Fr. Mersenne's church meetings at Paris. At the age of sixteen, Pascal presented a single piece of paper to one of Mersenne's meetings in June 1639. It contained a number of geometry theorems, including Pascal's mystic hexagon.

Desargues who is known to mathematicians through his projective geometry ideas and the well known enterprising scientist Galileo also lived almost in the same era. In December 1639 the Pascal family left Paris to live in Rouen where Étienne had been appointed as a tax collector for Upper Normandy. Hence Blaise started corresponding with Fr. Mersenne and so also with Fermat.

Mersenne was born in a small town in France in 1588. His parents were not wealthy, but they made sure he had a good education. He attended a Jesuit school, the same school that René Descartes had attended a few years earlier. Mersenne was expected to join the Catholic Church, as was common for men of his social class at the time. However, he was also eager to study further. On his way to Paris to study philosophy and theology, he stayed at a convent of the Order of the Minims. This brief experience touched him deeply, and he decided to join the order thus becoming a priest. Actually Mersenne completed his studies in the Sorbonne in 1611 and was ordained a priest in 1613. He remained a member of the Minims for the rest of his life.

One of his biographies states: "....The Minims realised that the biggest service he could give was through his books and they never asked any more of him".
By 1614 Mersenne was teaching philosophy and theology in the monasteries of the Order. Around this time he discovered and explored the curve known as the cycloid. It was during this period that his characteristic style of work began to take shape: the way he maintained links with scholars and exchanged ideas with them [RESONANCE] For instance when Mersenne heard about Galileo's findings on the motion of falling bodies, he set about verifying them himself. He conducted his own experiments and confirmed that Galileo's results were correct. He then wrote a letter to Galileo, praising his work and sharing his own findings. In fact Mersenne also helped to spread Galileo's work by publishing it in his own journal, the "Correspondence of Various Learned Men".

Mersenne was one of the first mathematicians to study the cycloid in detail. He published a book on the cycloid in 1634, in which he gave its mathematical definition and properties. He also showed how the cycloid could be used to solve a variety of problems in geometry and mechanics. Mersenne publicized his work on the cycloid through his correspondence with other mathematicians and scientists. While Galileo was one of the first mathematicians to study the cycloid and was interested in the curve because of its properties, such as its constant speed of rolling he could only be satified about the relation (it's area is three times that of the generating circle) by actually cutting the cycloidal arch and generating a circle by that material. So Mersenne was the one who used calculus to prove the relation. This and may other interesting mathematical facts were written concisely through postal correspondences.

Mersenne used to write about his number theoretic discoveries to Pascal and also Piere Fermat the amateur mathematician. In return he also received several such corespondences. For instance in 1640, Pascal wrote to Mersenne about his invention of the first mechanical calculator, the Pascaline. Mersenne was impressed by the Pascaline, and he helped to promote it to other scientists and mathematicians.

In 1637, Fermat sent Mersenne a letter in which he outlined his method for solving some Diophantine equations, sum of two squares and some facts about the result now known as Fermat's little theorem which is one of the most important results in number theory. In return in 1640, Mersenne published a book called "Cogitata physico-mathematica", which contained a number of Fermat's mathematical discoveries. This book helped to make Fermat's work known to a wider audience. Also in 1640 Fermat was asked by Frenicle to find a large perfect number, which led to a correspondence between the two men and this was also communicated to Mersenne. On the other hand Fermat and Pascal also corresponded on probability, as evidenced by a letter from Fermat to Pascal in which he discusses a problem of points, gambling.

**SOURCES**

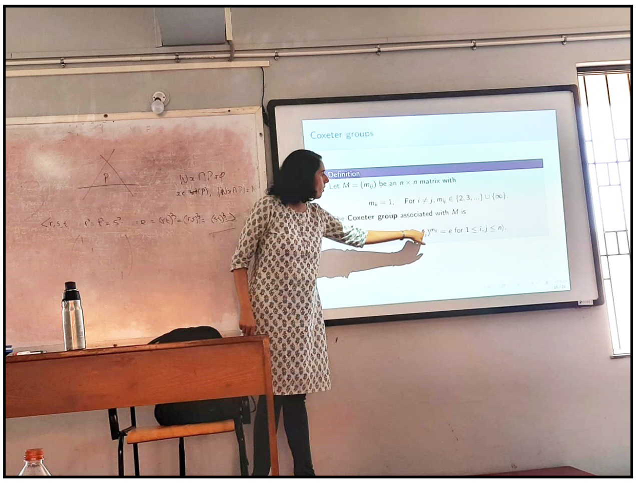
[1] Shailesh Shirali, Marin Mersenne 1588-1648, Resonance March 2013.

[2] C. Fletcher, A Reconstruction of the Frenicle-Fermat Correspondence of 1640, Historia Mathematica.

[3] Peter. M.L, Lecture Note on FERMAT AND PASCAL ON PROBABILITY(unpublished)

# MSI Round Up

There are currently various activities taking place at the Institute, and this column aims to report on them. Typically, the Newsletter is released in May or June to coincide with the May-12 initiative for women in Mathematics. However, this year's release was delayed for a couple of reasons. One of the reasons was that the women in mathematics programme happened on May-16 and we wanted to bring out the proceedings of the same. Also one of our mentors Prof. Krishna B. Athreya passed away and here we have constructed a sketch of his achievements and involvements with the Institute. Indeed he was like an extended family member to the members of the Institute.




*Prof. Tulsi Srinivasan* *MSI File Photo*

Prof. Tulsi Srinivasan, as part of the celebration for women in mathematics program delivered a lecture on the topic of Coxeter groups and related topology. During her lecture, she not only developed the relevant theories of arithmetic groups but also attempted to demonstrate a model for the reflections that were part of the motivations for the emergence of certain classes of Coxeter groups.

A little prior to that the 'Kishore Marathe School of Theoretical Physics and Computing' was inaugurated on 3rd January 2023. This is in memory of Professor Kishore Marathe an eminent mathematician who worked in the theoretical physics arena and one of the earlier persons who developed the theme of Physical Mathematics. During this a tribute to late Prof Kishore Marathe was read out. The message was sent by her daughter from the United states. Here is a part of the message:

*We'd like to begin by thanking Dr. & Mrs. Chintamani Kulkarni and family for their unwavering support and love, and for representing the Marathe family at our behest. I'm Arati Marathe, Prof. Marathe's daughter lending her words on behalf of Sanju Marathe (wife and mother), Ajay and Lina Marathe (son and daughter in law) and Leon LaSpina (son in law).*

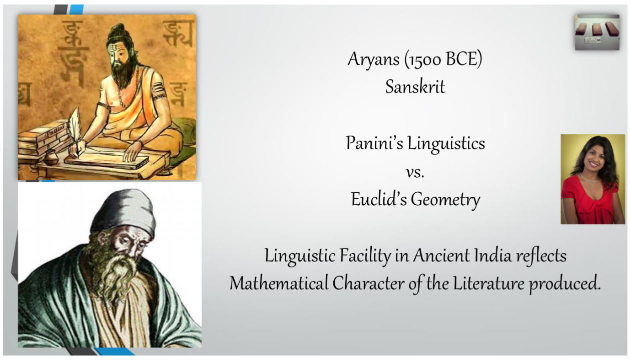
*I was dragging my feet quite a bit in writing our father's tribute. I suppose the finality of it blocked me from doing so. But then, like a beacon of light shining on us we received wonderful news from Prof. Venkatesh Tamraparni. Our father was supposed to be a keynote speaker here in Belagavi and a school building was now going to be inaugurated in his name. What an incredible honor bestowed upon him and our family. The Prof. Kishore Marathe School of Theoretical Physics. We were all bursting with excitement and thinking about how absolutely humbled Dad would've been....*

*Inauguration of Kishore Marathe School* *MSI File Photo*

The National Mathematics Day celebration got delayed due to some reason and so a commemorative lecture was held on 9th January 2023 by Professor Ken Ono, Marvin Rosenblum Professor of Mathematics at the University of Virginia. He spoke on the theme 'Remembering Ramanujan'. Truly it was a talk recalling his life and works. He also made some autobiographical remarks of himself as to how he got interested in Ramanujan's works.

Earlier on through the second wave of the pandemic MSI conducted several programmes online. The pandemic and the aftermath of the devastating natural calamity made us to combine the reports of both 2021 and 2022.

On 22nd December the birth anniversary of Srinivasa Ramanujan, Professor T. Venkatesh, Director MSIB, announced an immersed learning theme on Algebraic Topology. MSI has in one of its flagship programme "History of Mathematics Lecture series" (HoM Lectures) invited Dr Ambika Natarajan, Visiting Professor - CEBS, Mumbai to deliver its third HoM lectures entitled "The Art of Mathematics: Musings from History". Dr Ambika spoke on the Mathematics of ancient India and Eastern Asia in general. In fact she tried to bring out a balance between the two class of historians of mathematics, the ones who are zealously propagate the eurocentric viewpoint and the others who give priority to the Indo-centric vision.




*Screen Shot of History of Math Webinar* *MSI File Photo*

During May June 2021, Mathematical Sciences Institute conducted a month long summer school for High school students. Around 15 students got benefited by this programme. The resource persons Prof. T. Veena, Chidanand Badiger and Padma Raju gave lectures emphasizing on active learning methods.

A lecture series on Mathematical Finance was held on 3rd and 4th July 2021. The resource persons were Prof. J. V. Ramana Raju and Prof. T.Venkatesh, Director MSI. The topics covered were stochastic optimization and computer skills to tackle with financial data.

A major online event held in the August of 2021 was the symposium on the Topology of Lie Groups. The speakers included Professor Mahan Mj, Prof P. Sankaran and Prof Chidanand Badiger. In the first session classical Lie groups and related linear algebra and geometry was discussed. This session was carried out by Prof. Parameswaran Sankaran of the Chennai Mathematical Institute, Chennai. In the following two sessions Topological aspects of the hyperbolic plane and general algebraic topology were discussed. Prof. Mahan Maharaj of the Tata Institute of Fundamental Research, Mumbai and Mr Chidanand Badiger, Honarary Fellow, MSI and currently faculty at BMS College of Engineering, Bengaluru were the speakers for these two sessions. The theme of the last speaker was "Group actions and left invariant vector fields."

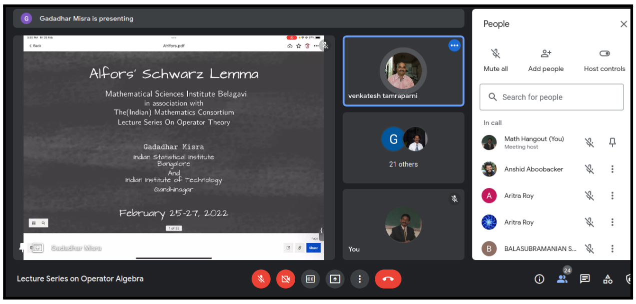
On September-11 2021, the Institute hosted the M.I Savadatti memorial webinar series on theoretical physics. The event featured several resource persons that included Professor Sunil Mukhi (Adjunct faculty CERN Geneva and ICTS-Bengaluru) and Prof. B. G. Mulimani former Vice Chancellor(BLDE University) and a physics professor. During his presentation, Prof. Mukhi discussed the mathematics and physics of the weak and strong forces, as well as the standard model of particle physics. The other speakers elaborated on various concepts that include symmetries, enumerative and tropical geometry and some intersections with mathematical physics.

Apart from mainstream discussions MSI also indulges into some interdisciplinary ideas. In November 2021, MSI had a beautiful online conversations called "Math and Music - A Dialogue" by Shri Sreedhar Kulkarni.

This programme had conversations interspersed with musical performances.

In December 2021 MSI celebrated the National Mathematics day with an online lecture entitled “The Prisoner-Hat Problem: An Introduction To Hamming codes”. The lecture was delivered by Prof Sharad Sane, Chennai Mathematical Institute.

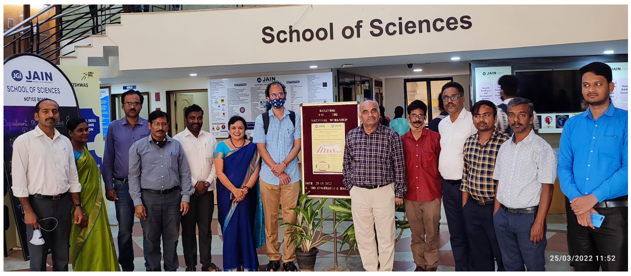
A lecture series on Operator Theory was organized during 25-27, 2022 by the Institute in association with IIT, Gandhinagar and Mathematics Consortium Pune. The three lecture delivered focussed on Complex Analysis, Ahlfor’s Schwarz lemma and related Geometry.




*Lecture Series(online) on Operator Theory* *MSI file photo*

In association with the Department of Data Analytics and Mathematical Sciences, JAIN (DEEMED-TO-BE) UNIVERSITY and MSI organized a Workshop on Random Walks & Financial Analytics during march 24-25, 2022. The resource persons for the workshop were Dr. T. Venkatesh Director Mathematical Sciences Institute Belagavi, Dr. Yogeshwaran D, Associate Professor, Indian Statistical Institute Bangalore, Dr. Anindya Goswami, Associate Professor IISER Pune and Prof. J. V. Ramana Raju, Department of Mathematics, Jain deemed to be University.

This programme was a joint effort by the respective teams of Jain Deemed-to-be University and the Mathematical Sciences Institute Belagavi. The programme was inaugurated by T. Venkatesh, Director, MSIB who also delivered the key note address. The one-day proceedings involved lectures on the basic probability themes emerging from the random walk model and related generalizations and also some very important modeling procedures used by the financial analytics industry. Firstly Dr. Yogeshwaran Dhandapani spoke about a 17th century problem posed by P. Fermat on the probabilistic modeling of a casino. Later Prof. T. Venkatesh threw light on the overall importance of mathematical finance. Professor Anindya Goswami spoke on the Black-Scholes-Merton model for derivatives pricing and on the CIR (Cox-Ingersoll-Ross) model. He also showed very basic simulations of the Brownian motion and the corresponding GBM process. Prof. Ramana Raju J V made an exposition of the probability models related to risk management analytics through concepts like Markowitz portfolio analysis, Bond immunization and other hedging tools. Around 150 participants including online viewers took part in the workshop.




*Workshop on Random Walks* *MSI File Photo*

The Institute organized an online lecture to give details about the Abel prize winner Denis Sullivan. The programme was joinly organized by IISER-TVM, Jain and Mathematical Sciences Institute Belagavi. This Abel prize lecture was delivered by Prof. Siddharth Gadgil of the Indian Institute of Science Bangalore. The event happened on April-29, 2022. During this lecture Prof. Siddharth who himself is a topologist and who had a stint at the Stony Brook University explained about the style and achivement of Sullivan’s work.

The year long thematic lecture series on “Geometry Topology & Combinatorics” was inaugurated by Prof. Harish Seshadri - IISc, Bengaluru. The event was held in online mode on 18th August 2022. He spoke on some scalar curvature aspects related to a special class of complex manifolds.

This programme was held in the school of sciences campus of JU. The speakers at the FDP were Prof. Apoorva Khare, IISc Bengaluru, Prof. Tulsi Srinivasan of APU-Bengaluru, Prof. N. S. N Sastry formerly of ISI-Bengaluru, Prof. T. Venkatesh, Director MSI, Belagavi, Prof. Pranav Pandit from the ICTS-Bengaluru and Prof. Arvind Ayyer again from IISc, Bengaluru. The event was held in hybrid mode with about 85 participants who were largely college teachers. A few research scholars were also present. 24 participants attended in offline mode and the rest viewed the lectures through zoom platform. The first speaker Prof. Apoorva Khare spoke on Blow up Polynomials of Graphs. His talk was algebraic in content explaining how discrete structures like Graphs were studied through certain classification themes to characterise them upto isomorphism when allowed to undergo blow ups of polynomials, exploiting algebraic notions such as Zariski density to ensure continuity under algebraic setting.

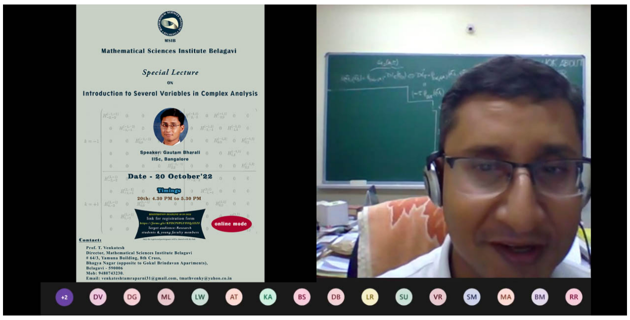

*FDP Session in progress* *MSI File Photo*

Prof. Tulsi Srinivasan of Azim Premji University spoke on Groups, Spaces and Dimensions and this talk was again under discrete algebraic setting with Cayley graphs as motivation to her talk through out. Prof. T. Venkatesh spoke on Projective geometry through algebraic preliminaries. Considering image processing of a camera, he delved into projective linear transformations and said that in this new geometry geometric types are preserved in the sense that points remain as points and so do lines. He tried to bring out the basic idea for projective geometry that is to view the underlying geometric object with regular points and points at infinity taking cue from algebraic geometry and so one has to start with the affine picture into the discourse and Projectivised version of them for the points going to infinity. Pranav Pandit from ICTS, dealt with Algebraic geometry themes tracing back the Euclid's axiomatic discription to the contemporary days of Groethendick's treatment. But the force was very concrete in convincing this high end mathematical abstraction. Also, his talk gave an idea how these themes motivated the problems of physics, especially physicists interest in string theory.

The second day proceedings witnessed lectures that saw applications into computer science and coding theory. Prof. Sastry dealt with the subject of prime numbers and associated algebraic structures including the finite projective planes. In his second talk Prof. Sastry gave a glimpse of applications to information theory and coding schemes. Prof. Arvind Ayyer spoke on "the matrix-tree theorem", an important theorem giving a formula for the number of spanning trees of a graph as a determinant he further explained historical evolution and the justifications to applications by theories propounded by Cayley and Kirchoff. A formal valedictory programme was presided by Dr. M. S. Reddy, Professor of Physics and Vice Principal, Jain School of Sciences, JU.


*FDP Session in progress* *MSI File Photo*

"Behaviour of Holomorphic Functions in Several Variables: The Role of Dimension", this was the title of the second lecture that was organised as part of the series: Geometry, Topology and Combinatorics. This part of the lecture series was delivered by Gautam Bharali, Professor IISc. The lecture emphasized the differences that one can observe when he or she compares complex analysis in one dimension and and then in $\mathbb{C}^n, n \geq 2$. Along the way complex differentiability, Hartog domains and other aspects were touched upon. The recent National Mathematics Day was celebrated with a special lecture, "Remembering Ramanujan". The commemorative lecture was not held on 22nd December, but was held later.



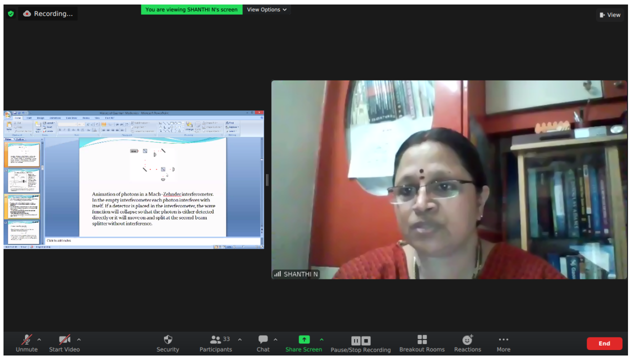

*Thematic Lecture by Gautam Bharali* *MSI File Photo*

**Foundation Day lecture series:**

To commemorate the foundation Day "Math Fest" a lecture series was held on January 23, 2023. Incidentally this day happens to be birth anniversary of David Hilbert. The speakers at the Fest were Aniruddha Sudarshan, Temple University Philadelphia, J.V. Ramana Raju, Jain University Bengaluru and Prof. T. Venkatesh Director, MSIB.


*Foundation Day Lecture Series* *MSI File Photo*

The first speaker Aniruddha Sudarshan who is a doctoral candidate at Temple University and also an alumnus of JGI spoke on Guassian integers, prime numbers and connection to zeta functions. He also elaborated on the patterns of prime numbers via PNT(Prime number theorem). The second speaker J. V. Ramana Raju spoke on spectral analysis arising out of eigen value problems and related applications in mathematical physics. The set-up he used is a Hilbert space of smooth functions on Euclidean and non-euclidean spaces and elaborated on physics connections. Earlier Professor T. Venkatesh delivered the key-note address and emphasized that youngsters pick up novel ideas emerging from premiere research institutes in India and abroad.




*Speical Lecture on Mathematical Physics Webinar by Dr N Shanti*

On March 23, 2023, the first major seminar through the Kishore Marathe School of Theoretical Physics and computing was organised. In this session Prof. Shanti N senior faculty(Head Department of Physics and Electronics, School of Sciences, Jain Deemed-to-be University) spoke on the mathematical aspects of Quantum Physics. This was a historically motivated talk bringing out the collaborative efforts of mathematicians and theoretical physicists.

# Prime Number Theorem and Some Anecdotes

This article elaborates on some of the letter correspondences that happened in the history of Mathematics in connection with the Prime Number Theorem(PNT).

The players involved in this game lived at different places and the period ranges from 16th century to 20th century! PNT is a statement about the distribution of prime numbers and hence part of number theory. Number theory has this peculiar character that the statements of some of the profound results are very easy to understand but their proofs are quite complicated. The first proof of the prime number theorem is one such that it is quite deep using complex function theory and some non-trivial analysis.


*picture credits - COMSOL BLOG*

The idea behind PNT arose in renaissance era. If one observes number of primes upto 100 and then between 100 to 200 and so on he or she is perplexed at the way primes are distributed, in the sense that, the counts don't give any visible pattern. We quote from the article by Kaneenika Sinha on prime numbers appearing in *Bhavana* article: "....And so it was in the year 1792, a young teenager by the name of Johann Carl Friedrich Gauss found himself in the possession of prime numbers up to 33 million. Gauss would go on to make many notable contributions to mathematics and physics that earned him the title "Princeps Mathematicorum" (Prince of Mathematics). By the time he turned 21, he had written the voluminous treatise Disquisitiones Arithmeticae, which did for number theory what Euclid's Elements did for mathematics in 300 BCE?

What Gauss analysed and observed led to his conjecture about an estimate for the number of primes upto x denoted as $\pi(x)$; and his conjecture was that $\pi(x)$ can be asymptotically computed by the function x/log(x). This is what later came to be known as PNT (Prime number theorem) proved for the first time in 1896.

It states that as x takes increasingly larger values, the prime counting function $\pi(x)$, defined as the number of primes p $\leq$ x, comes closer and closer to the value of x/log(x). i.e as x grows larger and larger one can see that the relative difference between the function $\pi(x)$, and x/log(x) approaches ZERO.

Gauss wrote these facts to his friend John Encko and this is how the world knows about his contributions to this area of research. It was much later that Hadamard and de la Vallee Poussin independently proved the PNT. Let us see the things in a chronological order. Prime numbers were objects of interest to the earlier mathematicians also Piere de Fermat an amateur mathematician in his post card correspondences with Mersenne and Pascal used to write about properties of certain types of primes. Later L. Euler was the first person to deal with the infinitude of primes in an analytic fashion. He in fact proved the infinitude by showing the divergence of the series $\sum(1/n)$.

By the mid-17th century log-tables were available and so Gauss with his computational nature seemed to have stumbled upon a pattern for the counting function $\pi(x)$. He studied the function x/log(x) and stated that a certain integral which is about x/log(x) is a good approximation to the prime counting function pi(x). [A technical term for the approximation is "asymptotically equal to"]. There is another mathematician, the Russian P. Chebyshev who has a role in this story about prime numbers.

Earlier towards solving this problem a major step was taken by Chebyshev in 1848 itself by viewing the function $\pi(x)$, as a summation, i.e a series. Thus rudiments of analytic number theory seem to have made their appearance here (although zeta type functions were considered by Euler also).

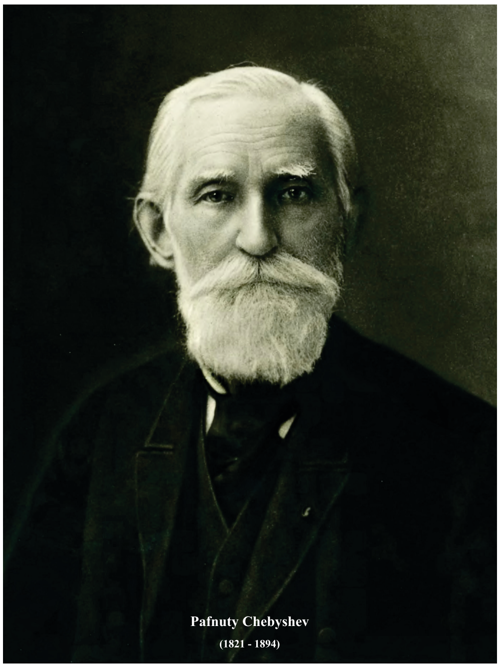
Chebyshev replaced the "elementary" prime counting function with the more sophisticated Lambda function because he believed that the Lambda function was a better way to measure the distribution of prime numbers. Through two special functions $\theta$(x) and $\psi$(x).

$$\theta(x) = \sum(log(p)), p \leq x$$

$$Tsi(x) = \sum(logp), p^m \leq x.$$

When Hadamard and de la Vallee Poussin finally proved this conjecture, they had to resort to the connection of prime distribution with the Riemann zeta function.

Eleven years after Chebyshev's work on prime numbers, Bernhard Riemann made a significant contribution to the field by introducing the Riemann zeta function, a complex-valued function that has a deep connection to the distribution of prime numbers. The Riemann zeta function (as is now being called) was first defined by Leonhard Euler more than a century before Riemann, but Riemann was the first to study its properties and connections with primes in detail.




*picture credits - wikipedia*

**Riemann's zeta function is defined as follows:**

$$\zeta(s) = 1 + 1/2^s + 1/3^s + 1/4^s + ....$$

where 's' is a complex number. The series converges for all values of 's' except for s = 1.

Riemann showed that the zeros of the Riemann zeta function are all located on the line s = $\frac{1}{2}$ + it, where 't' is a real number. He also conjectured that all of the non-trivial zeros (those with t not equal to 0) lie on this line. This conjecture, known as the Riemann hypothesis, is one of the most important unsolved problems in mathematics. The Riemann zeta function has been studied by mathematicians for over 150 years, and it continues to be a source of fascination and mystery. It is a powerful tool for studying the distribution of prime numbers apart from its influence to other areas of mathematics.

In the mid 20th century (1948 to be precise) Atle Selberg who was serving at Syracuse University published a paper that gave an elementary proof to PNT. Atle Selberg is regarded as one of the several brilliant mathematicians of the 20th century. He is famous for several novel theories including analytic and spectral aspects of number theory, rigidity theory the theory of automorphic forms, and on discrete groups. He received Fields medal in 1950 for his work on Riemann zeta function and trace formulae in number theory.

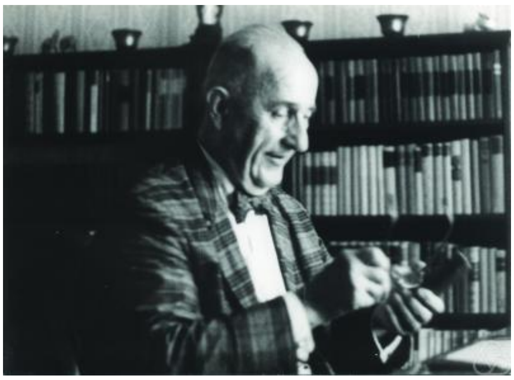

*picture credits - britannica.com*

Earlier Selberg defended his doctoral thesis in 1943 when he was at Oslo, Norway. The German invasion then stifed his career until Carl Ludwig Siegel encouraged him to apply for a 1-year position at the Institute for Advanced Studies in Princeton, USA, which was successful and where Selberg spent the academic year 1947-48. He was offered a one-year continuation at Princeton, but instead he chose to move to Syracuse. In this transition period, there was this famous letter correspondence with another giant in Number theory (and combinatorics) Paul Erdos. The elementary proof for the Prime number theorem came to light through these correspondences. In fact the ideas in Selberg's proof arose through the insights he obtained while studying the Riemann zeta function.

As we have stated earlier two other mathematicians proved the PNT in 1896, however the proofs were using Complex analytic methods. Hardy insisted that one needs an elementary proof, that is one which confines itself only to number theoretic and basic mathematical arguments.

Atle Selberg was in the transit between IAS, Princeton and Syracuse University when the whole exchange of ideas between him and Paul Erdos happened. He returned to Institute for Advanced Study, Princeton in 1949 as a permanent member. He was made Professor at the Institute's School of Mathematics and later was given the Emeritus Professorship.

In the 1940's, Atle Selberg worked on the theory of the Riemann zeta function and related problems concerning the distribution of prime numbers. The Riemann hypothesis states that all the non-trivial zeros of the Riemann zeta function lie on a certain line in the complex plane. Selberg showed that a positive proportion of these infinitely many zeros lie on this line. He also developed powerful new sieving methods and proved the prime number theorem, which had been sought for over 150 years. As mentioned earlier for his work, Selberg was awarded the Fields Medal in 1950.



*Carl Ludwig Siegel* *picture credits wikipedia.org*

# LEGENDS

## George Polya - a prolific Mathematician.

*George Polya* *Photo Courtesy Schoengeometry.com*

*George Polya was a prolific and brilliant mathematician who made important contributions in combinatorics, number theory, probability, complex analysis, and partial differential equations. Polya is considered as the father of mathematical problem solving according a prominent mathematician B. Sury [2].*

In this article we discuss his life and works in the legends section. His work gets connected to other themes touched upon in this issue namely crustallography and his advice to math-education.

Born in Budapest, Hungary to Jewish parents (who later got converted to Catholicism), Polya initially got trained as a lawyer and then he worked in an insurance company before he made a decision to switch to the profession of an educationist. He wanted to get into economics or statistics and he got appointed as a privatdozent at the University of Budapest. He then moved to Zurich where he worked alongside great intellectuals of the time including Hermann Weyl and Hurwitz. Interestingly while still as a student back in 1913 he conjectured to Szego a result concerning Fourier co-efficients!
From Zurich Polya moved to Brown University where he worked briefly (for two years) and then he finally settled down to Stanford University working there until retirement. In fact even after retiring he was active in the department until he was 90 years of age.

Polya contributed vastly in a variety of mathematical areas and allied fields. Below we have identified most of his well known contributions.

**Probability Theory:** In probability theory Polya looked at the Fourier transform of a probability measure showing in 1923 that it was a characteristic function. Even before that he wrote on the normal distribution and coined the term "central limit theorem" in 1920 which is now standard usage. In 1921 he proved his famous theorem on random walks on an integer lattice. In fact the name Random walks is due to Polya.

He considered a d-dimensional array of lattice points where a point moves to any of its neighbours with equal probability. He asked whether given an arbitrary point A in the lattice, a point executing a random walk starting from the origin would reach A with probability 1.

He showed that for d=1 and d=2 it is possible, but this is not possible in 3 or more dimensions. In a later work he looked at two points executing independent random walks and also at random walks satisfying the condition that the moving point never passed through the same lattice point twice.

His other contributions to probability: The continuity theorem for moments, stable distributions, the theory of contagion and exchangeable sequences of random variables, and the roots of random polynomials.

**Combinatorics:** One of the most influential areas of study for George Polya is combinatorial mathematics. Here his notable contribution is an application to chemistry. In particular he was interested in isomers and some enumerations therein. Combinatorics is the study of how many ways there are to do something, especially when the answer is too large or complicated to be figured out by just thinking about it. It also deals with the art of counting the number of possible arrangements of objects.

Polya's chief discovery was the enumeration of the isomers of a chemical compound, that is, the chemical compounds with different properties but the same numbers of each of their constituent elements. The problem had baffled chemists. Polya treated it abstractly as a problem in group theory and was able to obtain formulas that made the solution of specific problems relatively routine. He used symmetries geometrically as well as combinatorially using typical techniques like group theoretic ones. One such problem he tried was that of tiling a plane or space using a combination of regular shapes. These techniques proved to be helpful in physics also.

**Complex Analysis**

He proved that the circle of convergence of a power series is "usually" a natural boundary for the function-that is, a curve past which the sum of the series cannot be continued analytically. It is, in fact, always possible to change the signs of the coefficients in such a way that the new series cannot be contained outside the original circle of convergence. According to Fabry's famous gap theorem, the circle of convergence of a power series is a natural boundary if the density of zero coefficients is 1. Polya proved that no weaker condition will suffice for the same conclusion .He also extended this theorem in several ways and found analogs of Fabry's theorem for Dirichlet series, which have a more complex theory.

In the 1920's Polya systematized his methods for dealing with problems about power series. This very influential paper deals with densities of sequences of numbers, with convex sets and with entire functions of exponential type - that is, with functions analytic in the whole complex plane whose absolute values grow no faster than a constant multiple of some exponential function $e^{Az}$. Functions of this kind have proved widely applicable in physics, communication theory, and in other branches of mathematics.

Polya also contributed to many other topics in complex analysis, including the theory of conformal mapping and its extensions to three dimensions.

One of Polya's favorite topics was the connections between properties of an entire function and the set of zeros of polynomials that approximate that function. He and I. Schur introduced two classes (what one calls Polya-Schur or Laguerre - Polya functions)that are limits of polynomials that have either only real zeros or only real positive zeros. There are now many more applications, both in pure and applied mathematics than Polya himself envisaged, including, for example, the inversion theory of convolution transforms and the theory of interpolation by spline functions.

Polya devoted a great deal of attention to the question of how the behavior in the large of an analytic or meromorphic function affects the distribution of the zeros of the derivatives of the function. One of the simplest results (simplest to state, that is) is that when a function is meromorphic in the whole plane (has no singular points except for poles), the zeros of its successive derivatives become concentrated near the polygon whose points are equidistant from the two nearest poles.

The situation for entire functions is much more complex, and Polya conjectured a number of theorems that are only now becoming possible to prove.

**Mathematical Physics:**

Polya's contributions to mathematical physics consisted of developing methods for approximations such as the following

- The shape of an object can affect its physical properties in many ways. For example, the shape of a drum-head affects the sound of the drum. A drum head with a flatter shape will produce a lower-pitched sound than a drumhead with a more curved shape. The electrostatic capacitance of an object also depends on its shape. The capacitance is a measure of how much electrical charge an object can store. An object with a larger surface area will have a higher capacitance than an object with a smaller surface area.

- The mathematical equations that describe these physical properties can be very complex, and they are often impossible to solve exactly. In these cases, we need to use approximations to find solutions. There are many different ways to approximate solutions, and the best method to use depends on the specific problem. The methods, like his work in other fields, were subsequently developed further by others.

- Polya was interested in finding ways to estimate physical quantities, such as the electrostatic capacitance, torsional rigidity, and the lowest vibration frequency. These quantities are often difficult to calculate exactly, so Polya developed a method for estimating them using approximations.

- One of Polya's methods is to replace the given domain with a more symmetric one. A symmetric domain is one that looks the same from all sides, and it is often easier to calculate physical quantities for symmetric domains. For example, the area inside a circle is easier to calculate than the area inside a square.

- Once Polya has replaced the given domain with a more symmetric one, he can then calculate the physical quantities of interest for the symmetric domain. If he knows that symmetrization increases or decreases the quantity in which he is interested, he can then use this information to find an inequality for the other property.

- For example, Polya showed that the electrostatic capacitance of a circle is always greater than the electrostatic capacitance of a square with the same area. This is because the circle is more symmetric than the square, and symmetrization always increases the electrostatic capacitance. These type of problems are generically referred to as isoperimetric problems.

One of the earliest successes of this technique was a simple proof of Rayleigh's conjecture that a circular membrane has the lowest vibration frequency (that is, the smallest eigenvalue of the corresponding differential equation) among all membranes of a specified area. For different problems, different kinds of symmetrization are needed.

Many physical quantities that are determined as the solutions of extremal problems can be estimated by making appropriate changes of variable, a technique known as "transplantation". Polya exploited this technique in a long paper in collaboration with M. Schiffer . His interest in approximations also led him to contribute to the theory of finite differences.

**Achievements:**

In 1933 Pólya was awarded a second Rockefeller Fellowship. In addition to a prodigious lifetime output of more than 250 papers, Polya in 1945 wrote "How to Solve It," which explains in non-technical terms how to think about invention, discovery, creativity and analysis. The book has been translated into 15 languages and has sold more than 1 million copies, making it one of the most widely circulated mathematics books in history.

Further he received the Wolf prize in 1954, National medal of science in 1966, the Leroy P. Steele Prize for Lifetime Achievement in 1983, the Bolyai Prize in 1984 to enlist a few major honors.

**Teaching of Mathematics:**

For teachers, Polya seems to have given certain commandments.

**Here is a gist of the same.**

Polya argues that the goal of education should be to teach students how to think, not just what to think. This means that teachers should not just be dispensers of information, but should also be facilitators of learning. They should help students to develop the skills and habits of mind that they need to think for themselves.

As one can see with his advice to students he emphasizes the importance of knowledge and useful attitudes. Knowledge is essential for critical thinking, but it is not enough. Students also need to be open-minded, willing to consider different perspectives, and able to think creatively.

On the art of teaching he says Teaching obviously has much in common with the theatrical art. For instance, you have to present to your class a proof which you know thoroughly having presented it already so many times in former years in the same course. You really cannot be excited about the proof - but, please, do not show that to your class; if you appear bored, the whole class will be bored. Pretend to be excited about the proof when you start it, pretend to have bright ideas when you proceed, pretend to be surprised and elated when the proof ends. You should do a little acting for the sake of your students who may learn, occasionally, more from your attitudes than from the subject matter presented...... Now and then, teaching may approach poetry, and now and then it may approach profanity.

He further notes, nothing is too good or too bad, too poetical or too trivial to clarify your abstractions.

As Montaigne put it: The truth is such a great thing that we should not disdain any means that could lead to it. Therefore, if the spirit moves you to be a little poetical or a little profane in your class, do not have the wrong kind of inhibition.

**About knowing:** Know about the ways of learning: the best way to learn anything is to discover it by yourself.

Give them not only information, but “know-how”, attitudes of mind, the habit of methodical work Look out for such features of the problem at hand as may be useful in solving the problems to come-try to disclose the general pattern that lies behind the present concrete situation.

One of the very famous quotes of Polya is: “If you can’t solve a problem, then there is an easier problem you can solve: find it”.

Thus Polya had some distinctive suggestions to budding students and teachers of mathematics.

Pólya believed that the best way to learn science and mathematics is to start with problems. He argued that students should be given problems to solve, and then they should be helped to develop the skills and knowledge they need to solve those problems. He also believed that it is important to focus on the process of problem solving, rather than just on the final answer. He argued that students should be taught how to think about problems, how to break them down into smaller steps, and how to use different strategies to solve them. He used to say that it is important to encourage students to be creative when solving problems. He argued that students should be given the freedom to experiment and try different approaches, and that they should not be afraid to make mistakes.

Also Pólya believed that it is important to provide students with feedback on their problem-solving efforts. He argued that feedback can help students to identify their strengths and weaknesses, and to learn from their mistakes. He often argued that students are more likely to learn if they are engaged and interested in the material.

Here are some excerpts of his writings in various places

- The aim of heuristic is to study the methods and rules of discovery and invention..... Heuristic, as an adjective, means 'serving to discover'..... its purpose is to discover the solution of the present problem,... What is good education? Systematically giving opportunity to the student to discover things by himself.

- Mathematics in the primary schools has a good and narrow aim and that is pretty clear in the primary schools..... However, we have a higher aim. We wish to develop all the resources of the growing child. And the part that mathematics plays is mostly about thinking. Mathematics is a good school of thinking. But what is thinking? The thinking that you can learn in mathematics is, for instance, to handle abstractions. Mathematics is about numbers. Numbers are an abstraction. When we solve a practical problem, then from this practical problem we must first make an abstract problem.....But I think there is one point which is even more important. Mathematics, you see, is not a spectator sport. To understand mathematics means to be able to do mathematics. And what does it mean doing mathematics? In the first place it means to be able to solve mathematical problems.

- Teaching is not a science; it is an art. If teaching were a science there would be a best way of teaching and everyone would have to teach like that. Since teaching is not a science, there is great latitude and much possibility for personal differences.... let me tell you what my idea of teaching is. Perhaps the first point, which is widely accepted, is that teaching must be active, or rather active learning.... the main point in mathematics teaching is to develop the tactics of problem solving.

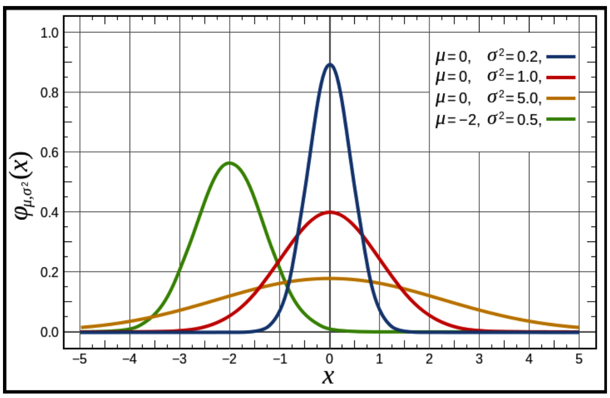
**SOURCES:**

1] Shailesh Sirali, George Polya and Problem solving: An appreciation Resonance April 2014

2] B.Sury, Article in a Box on George Polya, Resonance April 2014

3] George Polya - A Biographical Memoir, R.P Boas, National Academy of Science, Washngton.

4] https://mathshistory.st-andrews.ac.uk/Biographies /Polya/




*Gaussian Distribution*

# MATH EDUCATION

## On Teaching of Abstract Mathematics

There have been recommendations coming from policy making institutions world over with regard to radical changes like new kinds of teaching strategies and bringing in technology aids in the teaching of Mathematics.
In this article we try to summarize the thoughts of some of the national decision makers. An analysis of the same seems to be significant especially for K-12 teachers keeping in mind the abysmally lowered enthusiasm towards the subject of Mathematics.

It has been seen that emphasis is laid on activity based learning and on thought provoking "try and discuss" segments in the curriculum. Also wherever possible flow charts and simulations are being recommended to understand the concepts especially where mathematical abstraction is involved.

Another commonly stated fact is that teachers at the school level need to be 'adequately trained' to make math interesting.It is important to understand the gist of this aspect - Teachers need to acquire "mathematical common sense". The elegance and power of mathematical reasoning needs to be communicated drawing analogy from real life situations. Also it is of utmost importance that teachers teaching at high school grades need to take an extra effort and try to connect material taught in schools to the one seen in freshman courses in college. For instance topics like 'limit concept', 'trigonometric ratios' have to be dealt with a lot of care.

Looking at ourselves and introspecting our teaching strategies leads to the fundamental question as to what are we conveying to students. Especially when it comes to abstract mathematics involving sets, binary operations and various structures, one must believe and convey that it is about ideas and patterns that make a universal aesthetic appeal, rather than a collection of motiveless rules. The universal aesthetic sense leads to ideas that can build confidence in problem solving ability in varied situations .

Finally to end, we look at words of wisdom - A quote by Hyman Bass[1] points out that you do not learn culinary art by eating out at fancy restaurants, you do not learn how to sing opera by attending performances, and you do not learn how to play tennis by watching the US-Open, you actually learn by doing. So a workbook culture is required where we as educators can supervise the practice of mathematics, done through problem solving and proof writing through mathematical reasoning.

Since we are focussing on active learning as opposed to passive learning we look at Inquiry based learning- IBL.

IBL is a form of active learning in which students are given a carefully scaffolded sequence of mathematical tasks and are asked to solve and make sense of them, working individually or in groups. In this article, we describe the core principles of IBL and provide three specific but representative examples of IBL classrooms from our perspective: upper-division

proof-based mathematics courses, the calculus sequence, and mathematics for elementary teachers. Active, student-centered pedagogies such as IBL exist

[1] [1] Amy Cohen,Steven Krantz: Two Reactions to the mathematical education of teachers, AMS Notices Vol 48, #9, 2001

in a dynamic landscape, so describing teaching methods that are constantly evolving is a challenging and slippery task. We are still developing and trying to understand the variety of IBL methods, when and where they are applicable, and identifying best practices. Here we share some of the commonly used examples and core ideas that drive instructor decisions.

The philosophy of IBL-based pedagogy rests on two pillars. One is **Deep engagement** which means that students are actively and intentionally working on challenging mathematics problems. They do a significant portion of the development of mathematical ideas, which are more sophisticated than rote skill-level exercises. Typically, students do not know the answer or method ahead of time, and the questions generally require grappling with mathematical ideas before arriving at a solution to the problem. In a calculus course for instance, students are presented with open-ended problems that require them to think critically and creatively about the concepts they are learning.

**Collaboration** is the process of working together to achieve a common goal. In mathematics, collaboration can help students to better understand concepts, to develop their problem-solving skills, and to learn how to communicate their ideas effectively.

Collaboration can take other forms besides group work. For example, in an upper-division course with a focus on proof, students often present their proofs to the entire class. The class peer-reviews the proof, discussing its features such as validity, techniques, and coherence. Hence, class discussion is a class-wide collaboration, moderated by the instructor. In this case the class works together to validate and understand the meaning of proof.

The many varieties of inquiry-based learning (IBL) are a natural result of the diverse and everchanging landscape of college mathematics education. Factors such as class size, students' prior experiences, course topic, and instructor's level of skill and experience all significantly affect how IBL is implemented.
Instructors must make many decisions about the structure of an IBL class, such as how much time to allocate to group work, how to facilitate student discussions, and how to assess student learning. The varied and disparate factors that influence these decisions have given rise to a great variety of IBL approaches.

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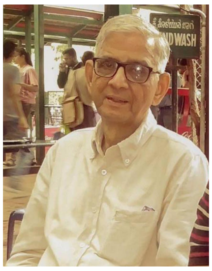
# TRIBUTES

## K. B. Athreya a dear mathematician passes away



*Krishna B. Athreya, Professor at Iowa State University a probabilist, breathed his last at his home in Ames, Iowa, on March 24, 2023, from complications of Multiple System Atrophy (MSA), a rare neurodegenerative disease. He passed away surrounded by loved ones, for whom his presence and memory are a blessing.*

Professor Athreya's last position was Emeritus faculty in mathematics and statistics at Iowa State University, Ames, Iowa, USA. His areas of research were probability theory, stochastic processes and mathematical analysis. He enjoyed teaching mathematics at all levels. He was very fond of Indian Classical music.

His dear ones (his children) explain: "Krishna, or KBA, or Appa (father) - meant so much to so many. He loved mathematics, music, travel, and Iowa State University. In the last year of his life, even when he could not stand, walk, write, or speak much on his own, he would still remember a theorem or corollary and surprise us with his insight. He loved going to Heartland Senior Center, where the staff greeted him with affection and made him his favorite peanut butter and jelly sandwich, and where he could talk about probability theory between rounds of Uno. He loved riding the HIRTA bus home to our mother Rani".

He met each challenge MSA threw at him. When he could no longer take walks around Ada Hayden Park, he would move with his walker. When that wasn't possible, they would wheel him in his wheelchair, sometimes in twenty-degree weather, and he would remark on a birdcall or sunset over the lake. When he could no longer easily get into a wheelchair, he would watch the maple tree outside his room and give his hands a little squeeze to say good morning. And no matter what he faced, on days when he couldn't swallow, or breathe, or had convulsive hiccups——they would ask him how he was, and his answer was always the same: "Not too bad."

His story is part of the tapestry of the American immigrant experience. Born one of 14 children and raised in the hamlet of Pattamadai in Tamil Nadu, India, he was barely eight years old when he endured the impoverishing death of his own father, a school principal who taught King Lear in their remote Indian village. Years later, he would make sure we understood that others' generosity had made the difference for him between survival and destitution. His mother, Narayani, knew that educating her children was the only way out.

Moving on to his academic acievements, KBA received his PhD at Stanford University under the supervision of Samuel Karlin. Soon after Athreya got his degree, he and Karlin worked on Branching processes in random environments. Two papers were published in the Annals of Mathematical Statistics in 1971. After these two papers several of his papers contained novel ideas which continue to inspire young students of probability even to this day. For example "A new approach to the limit theory of recurrent Markov chains," which appeared in the Transactions of the American Math Society, a paper that was written with Peter Ney in 1978. This is an area about extending Markov space on a general space rather then a routine discrete state space. Random Logistic maps, Large deviations for branching processes are few other themes he worked on. KBA was surrounded by brilliant and engaging colleagues from all over the world, and an everreplenishing supply of open problems, many of which he both posed and answered: on renewal processes; branching processes in random environments; regenerative Markov chains; and as recently as 2021, extrema of Luroth digits. His book with Peter Ney on branching processes remains the definitive treatment of the topic. His doctoral students carry on this legacy. He was committed to making mathematics accessible. His text Measure Theory and Probability, written with Soumendra Lahiri, is a comprehensive graduate-level introduction to the subject. He delighted in sharing drafts with his family and generating instructive, tractable problems for its chapters.

Back in India he held positions at premiere science campuses like the Indian Institute of Science and TIFR-CAM (Center for Applicable Mathematics). As per the mathematics genealogy project Professor Athreya had two students from India namely C. E. Veni Madhavan and C.R Pranesachar both of whom later on served on the faculty of the Insian Institute of Science.

KBA liked to joke that no one could have predicted his success—he failed math tests in school, and it wasn't until a teacher helped him think about mathematics more intuitively that he came to grasp it. After Loyola College, Madras, he trekked to the Indian Statistical Institute, and then, through the largesse of the Fullbright Fellowship, to Stanford. He completed his thesis there, on limit theorems for Markov branching processes, the start of a long and productive career. He held faculty positions at the University of Wisconsin, Madison; the Indian Institute of Science; Cornell University; and, of course, Iowa State University, his home for nearly half a century.

We remember his journeys across the Indian academic scene where he would combine his mathematical expositions with his talk about Hindustani music (about which we shall say later). He would often travel to towns in India to give lectures on mathematics to high schoolers. For decades, he would journey to India each year, and he would showcase his bicultural perspective, fluency in multiple languages, and attachment to the rhythm, rituals, and history of his birthplace. No description of his life would be complete without mentioning his other great love, music: Indian classical music, both Carnatic and Hindustani, and western classical, especially the Baroque and classical periods (he had a very soft spot for Beethoven's Sixth Symphony, the Pastoral, and held on to an old, hissing cassette recording of it by Bruno Walter and the Columbia Orchestra). His house was filled with music. He taught each of his children to identify Indian musical scales, called ragas, and would travel far and wide to concerts. His collection of live recordings is as rich and eclectic as his tastes: the Carnatic vocalist MD Ramanathan, the sitarist Shahid Pervez, the violinist Yehudi Menuhin, the flautist Shashank, and his own sister-in-law Gargee Siddhant Dutta. In Ames, he would conduct "Raga Sessions" at his home—evenings of music for anyone who was interested, in which he shared his knowledge and the contemplative peace that music brought him.

Athreya has left an impressive mathematical legacy that will enrich a student's life and research if he or she has the time to read it. We at MSI met KBA for the first time in 2004 at a special lecture organised at the behest of Vittal Rao from IISc and since then became academic and family friends with him. It is sad that there will be no more work coming from him, no more thoughts of wisdom emanating from a fatherlike figure but I hope others who read this will be inspired to continue his work.

## K. R. Parthasarathy

Kalyanapuram Rangachari Parthasarathy, a renowned mathematician and probabilist, passed away on June 14 in New Delhi at the age of 86. Known to his peers simply as KRP, he made profound and far-reaching contributions to a wide range of mathematical fields, including probability, quantum probability, graph theory, linear algebra, statistics, and more. His passing is a great loss to the Indian mathematical community and to the world of mathematics as a whole.

KRP as he was fondly called was born in Madras (now Chennai) in 1936. He received his BA (Honours) degree in Mathematics from Vivekananda College in 1956. In the same year, he moved to the Indian Statistical Institute (ISI) in Calcutta, where he studied under C.R. Rao, D. Basu, Raghu Raj Bahadur, and Radha Govind Laha. He also had the opportunity to interact with other brilliant mathematicians, such as Ranga Rao, V.S. Varadarajan, and Raghu Varadhan.

*K. R. Parthasarathy* *Photo credits Bhavana*

During his time at ISI, Parthasarathy was part of a group of mathematicians known as the "famous four." This group was known for their deep and broad education in mathematics, especially in the areas of ergodic theory, limit theorems, topological groups, non-Fourier methods, Lie algebras, and information theory. This education was a catalyst for Parthasarathy's future career in mathematics and quantum probability.

In 1962, Parthasarathy received his PhD from ISI Calcutta under the supervision of C.R. Rao. After his PhD, he spent a year at the Steklov Institute in Moscow, where he attended seminar series by some of the leading mathematicians of the time, including Andrey Kolmogorov, Efim Dynkin, and Israel Gelfand. These experiences had a profound influence on Parthasarathy's subsequent work. While at ISI, Parthasarathy married his wife Shyama. The famous botanist T.A. Davis was working with J.B.S. Haldane, who was also a faculty member at ISI, and he graciously found a modest campus accommodation for Parthasarathy and his wife.

Shortly after, Parthasarathy left for Sheffield at the suggestion of Dr. Varadarajan. It was there that Eugene Lukacs saw Parthasarathy's handwritten notes on probability measures and weak convergence in (mostly, separable) metric spaces. Academic Press published Parthasarathy's classic book, Probability Measures on Metric Spaces, in 1967.

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Nearly sixty years after its publication, it is still regarded as one of the main foundations for the construction of the elegant one-dimensional empirical process theory, which has become a grand unifier of almost all of asymptotic theory. It has received over 4,400 citations so far.

Parthasarathy authored many other books and monographs on quantum probability and SDEs on abstract spaces, which are also regarded as first-rate contributions. Some of his most notable works include:

- A 1984 paper with Robin Hudson on quantum Itô formulas, which has been cited over 1,600 times.
- A 1963 article with Ranga Rao and Raghu Varadhan on probabilities on locally compact Abelian groups.
- A 1967 article with Ranga Rao and Varadarajan on representations of Lie groups.
- A 2000 paper with Rajendra Bhatia on positive definite functions.
- A 1992 book on quantum stochastic calculus, which has been cited over 1,700 times.

Parthasarathy also said that he was inspired to consider positive definite kernels after reading a comment made by Joe Doob that the proof of the Lévy-Khintchine formula for the characteristic function of an infinitely divisible distribution on the real line rests entirely on positive definite functions.

In addition to his books and monographs, Parthasarathy also published numerous papers with Kalyan Sinha on operator theory and quantum diffusions, martingales and Markov processes. He also wrote many other papers on graph theory, infinite divisibility, extreme points of convex sets, tomography, transmission rate of information, C algebras, and several other interesting topics.∗

Following his years at Sheffield, Parthasarathy moved to Manchester and served as faculty of statistics for several years there. He and his family returned to India around 1967 at the initiative of S.S. Shrikhande. He joined the CASM at Bombay (now Mumbai).

C.R. Rao had already moved to Delhi from Calcutta (Kolkata), and in 1973, he suggested to Parthasarathy that he move to Delhi as well. Rao very much wished that Parthasarathy join the Delhi center of the ISI, which was due to be opened soon. Parthasarathy accepted Rao's suggestion and joined the IIT Delhi. One year later, in 1974, Indira Gandhi inaugurated the Delhi center of the ISI, where Parthasarathy was Distinguished Professor until his retirement in 1996.

Among his many distinguished students are K. Balasubramanian, B.V.R. Bhat, Rajendra Bhatia, and Inder Rana. He was Distinguished Professor Emeritus at ISI Delhi at the time of his passing.

He received numerous awards starting from the Shanti Swarup Bhatnagar Prize in Mathematical Sciences (1977), He was elected Fellow of the Indian National Science Academy, New Delhi.

Besides he was invited to lecture at the International Congress of Mathematicians, Zurich, on the theme Quantum stochastic calculus.

Parthasarathy was dedicated to the passionate pursuit of mathematics and probability all his life, he used to suggest that fresh PhD's should talk to each other, generate ideas, conduct seminars and spread ideas by always writing notes on the seminar topics. That was his model for spreading world-class mathematics in India on a large scal; he did not have other ambitions. But he did have other interests. He loved classic English literature and Indian classical music. He had a great sense of humor. He was a true and unfeigned academician, an esteemed representative of the best and golden days of the ISI, the days that still live in India's romantic nostalgia.

He is survived by his wife Shyama and their two sons.

## Tribute to CR Rao Father of Modern statistics

C.R. Rao, a world-famous statistician who had been teaching at the University at Buffalo since 2010, died on August 22 at the age of 102. He was a pioneer in the field of statistics and helped to lay the foundation for modern statistical methods. Earlier this year, he was awarded the 2023 International Prize in Statistics, which is considered to be the Nobel Prize for statistics.

"Dr. Rao was a brilliant statistician who revolutionized his field," said President of UB Satish K. Tripathi, who has two master's degrees in statistics. "He was also a rare person who combined genius with humility. It was an honor to have him as a member of our faculty, and his time at UB was the culmination of an amazing career that spanned 80 years." he said

*C. R. Rao* *Photo credits The Hindu*

While our academic community mourns his loss, we are comforted by the knowledge that his legacy will live on in the researchers around the world, including the many people he mentored, who are using his innovative theorems and visionary contributions to advance the field to which he devoted his life.

Talking about his research, the first, known as the Cramer-Rao lower bound, provides a benchmark for judging the accuracy of estimation methods. The second, known as the Rao-Blackwell theorem, shows how to improve the accuracy of an estimate and the third major contribution was to pioneer the field of information geometry, which has found applications in a wide range of fields, including particle physics, radar, antenna technology, and artificial intelligence.

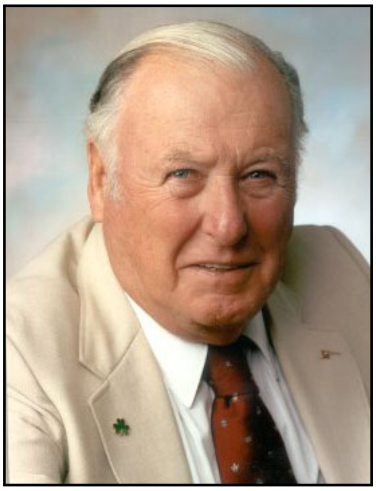
Dr. Rao spent most of his career at the Indian Statistical Institute, where he made three groundbreaking discoveries by the age of 25. These discoveries helped to establish statistics as a separate field of study from mathematics.
In 1979, Dr. Rao retired from ISI and moved to the United States. He taught at the University of Pittsburgh and Pennsylvania State University before coming to UB in 2010 as a research professor in the Department of Biostatistics.

During an interview at his 100th birthday he said "In my lifetime, I have seen statistics grow into a strong independent field of study based on mathematical - and more recently computational - tools. Its importance has spread across numerous areas, such as business, economics, health and medicine, banking, management, and the physical, natural and social sciences." In announcing Rao's receipt of the International Prize in Statistics, the International Prize in Statistics Foundation noted that "Rao's work more than 75 years ago continues to exert a profound influence on science," adding that three fundamental results Rao published in 1945 "paved the way for the modern field of statistics and provided statistical tools heavily used in science today." In addition to the International Prize in Statistics, Rao was the recipient of numerous other prestigious awards, including the India Science Award in 2014 and the U.S. National Medal of Science, presented by President George W. Bush in 2002. In 2013, he was nominated for the Nobel Peace Prize.In 2002, Rao established the C.R. Rao Advanced Institute of Mathematics, Statistics and Computer Science in Hyderabad, India.

He authored 476 research papers - 201 between 1940 and 1980 in India, and 275 between 1980 and 2010 in the United States. He has written 15 books, including leading textbooks in the field.

# REFLECTIONS

## Reflections:– An address by James W Cooley to the Society for Industrial and Applied Mathematics



*James William Cooley* *ethw.org*

James W Cooley together with John Tukey built one of the famous algorithms used worldwide. However Cooley brings out the historical anecdotes and the many people who played a role in this very important achievement. Cooley at the SIAM meeting said that the purpose of this meeting is to bring together pioneers whose vision and research have made major contributions to specific areas of the computing field. He also emphasised that the purpose of the meeting is to honor people who have made significant contributions to the field of computing. Cooley then goes on to say that they have been more successful in paying attention to the vision of others. The history of the FFT is fascinating and full of ironies and enigmas he said and believed that the FFT is a good example of the importance of professional societies. What follows is an excerpt of his SIAM address.

My involvement with the FFT algorithm, or algorithms as we should probably say, started when Dick Garwin came to the computing center of the new IBM Watson Research Center some time in 1963 with a few lines of notes he made while with John Tukey at a meeting of President Kennedy's Scientific Advisory Committee. where they were both members. In the context of Fourier Theory, John Tukey showed that, if N, the number of terms in a Fourier series is a composite, i.e N = ab, then the series can be expressed as an a-term series of subseries of b terms each, If one were computing all values of the series, this would reduce the number of operations from $N^2$ to $N(a + b)$. Tukey also said that if this were iterated, the number of operations could be reduced from N to NlogN. Garwin not only had the insight to see the importance of this idea but also had the drive to pursue its development and publication. Dick told me that he had an important problem of determining the periodicities of the spin orientations in a 3-D crystal of He .I found out later that he was also trying to find ways of improving the ability to do remote seismic monitoring in order to facilitate agreement with Russia on a nuclear test ban and to improve our capability for long range acoustic detection of submarines. Like many others, I did not see the significance in this improvement and put the job on a back burner while I continued with some research I considered more important. However, I was told of Dick Garwin's reputation and, prodded by his occasional telephone calls (some of them to my manager), I produced a 3-dimensional FFT program. I put some effort into designing the algorithm so as to save storage and addressing by over-writing data and I spent some time working out a 3-dimensional indexing scheme that was combined with the indexing within the algorithm Garwin publicized the program at first by personal contacts, producing a small but increasing stream of requests for copies. I did a write-up and a version for a program library, but did not plan publishing right away. I gave a talk on the algorithm in one of a series of seminars in our mathematics department. Ken lverson and Adin Falkoff, the developers of APL, participated and Howard Smith, a member of the APL group, put the algorithm in APL when it was only a language for defining processes and before it was implemented on any machine. This gave the algorithm a thorough working over at the seminar. Another participant was Frank Thomas, A mathematically-inclined patent attorney, who kept good contacts in the mathematics department. He suggested that there were patent possibilities and a meeting was called to decide what to do with it. It was decided that the algorithm should be put in the public domain and that this should be done by having Sam Winograd and Ray Miller design a device that could carry out the computation. My part of the strategy was to to publish a paper with a footnote mentioning Miller and Winograd and their device. I sent my draft copy to John Tukey, asking him to be co-author. He made some changes and emendations, and added a few references to F. Yates, G. E. P. Box, and I. J. Good. Next came the task of getting it published as quickly as possible. I offered it to Mathematics of Computarion by sending it to Eugene Isaccson at the Courant Institute of Mathematical Sciences, where I had worked before coming to IBM. I do not know how important my acquaintance with Eugene was or what effect it had on getting the paper published quickly. In any case, it appeared 8 months after submission in the April, 1965 issue.

I found out later about an excellent paper by Gordon Sande, a very bright statistics student of Tukey's, who was exposed to the factorization idea in one of Tukey's courses. He carried the subject further, showing how it could be used to reduce computation in covariance calculations. After hearing about our paper going out to Mathematics of Computation, he did not publish his in its original form. However, he published several other excellent papers one of which showed that the new algorithm was not only faster but more accurate. His form of the FFT is now known as the Sande-Tukey algorithm.

Another result of Dick Garwin's efforts was a seminar run at the LBM Watson Research Center to publicize the algorithm and familiarize IBMers with it. For this, two very capable statisticians, Peter D. Welch and Peter A. W. Lewis, joined me in writing a thick research report describing the algorithm and developing some theory and applications. The three of us then published a series of papers on applications of the FFT. These papers elaborated on the theory of the discrete Fourier transfrom and showed how standard numerical methods should be revised as a result of the economy in the use of the FFT. These included methods for digital filtering and spectral analysis.

**The IEEE ASSP Digital Signal Processing Committee**

The next level of activity came with contact with the speech and signal processing people at MIT-notably Thomas Stockham, Charles Rader, Alan Oppenheim, Charles Rabiner-all of whom have gone on to become highly renowned people in digital signal processing. They had developed digital methods for processing speech, music, and images. The very great obstacle to making their methods feasible was the amount of computing required. This was the first really impressive evidence to me of the importance of the FFT. I was invited to join them and others on the Digital Signal Processing Committee of the IEEE Acoustics Speech and Signal Processing Society.

This committee ran the now famous Arden House Workshops on the FFT in 1967 and in 1969. These were unique in several respects. One was that they collected people from many different disciplines: there were heart surgeons, statisticians, geologists, university professors, oceanographers, just to name a few. The common interest was in the use of the FFT algorithms and every one of the approximately 100 attending had something useful to say in his presentation. Another thing that was unique was that work was really done. People got together to formulate and work out solutions to problems. An example was where Norman Brenner, then of MIT, designed a program that computed the FFT of a sequence of interferometer data of 512,000 elements, which was larger than available high-speed storage, He did this for Mme. Connes, of the University of Paris, who returned home to perform a monumental calculation of the infra-red spectra of the planets which has become a standard reference book. Others worked out algorithms for data with special symmetries.

**Recent Early History of the FFT**

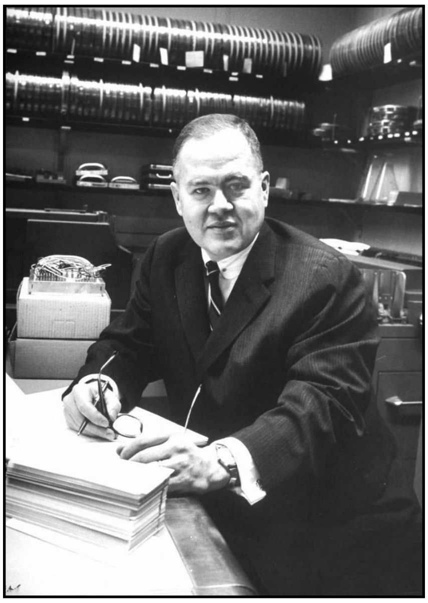
Meanwhile, back at the research center, I started learning the history of the FFT. Dick Garwin questioned his colleague, Professor L. H. Thomas of the Watson Scientific Laboratory of Columbia University, who had an office next to his. Thomas responded by showing a paper he published in 1963. His paper describes a large Fourier series calculation he did in 1948 on IBM punched card machines: a tabulator and a multiplying punch. He said that he simply went to the library and looked up a method. He found a book by Karl Stumpff that was a cook-book of methods for Fourier transforms of various sizes. Most of these used the symmetries and trigonometric function identities to reduce computations by a constant factor. In a very few places Stumpff showed how to obtain larger transforms from smaller ones, and then left it to the reader to generalize. Thomas made a generalization that used mutually prime factors and got a very efficient algorithm for his calculation. The algorithms of Good and Thomas mentioned above have some very favorable properties, but the constraint that the factors are mutually prime does not give a number of operations proportional to or as low as N IogN. Tukey's form of the algorithm, with repeated factors, has the great advantage that a computer program need only contain instructions for the algorithm for the common factor. Indexed loops repeat this basic calculation and permit one to iterate up to an arbitrarily high N, limited only by time and storage.

The credit for what 1 would consider the first FFT- a computer program implementing this iterative procedure and really giving the N IogN timing, should go to Philip Rudnick of the Scripps Institution of Oceanography in San Diego, California. He wrote to me right after the publication of the 1965 paper to say that he had programmed the radix 2 algorithm using a method published by Danielson and Lanczos in 1942 in the Journal of the Frankin Insitute, a journal of great repute which publishes articles in all areas of science, but which did not enjoy a wide circulation among numerical analysts. Rudnick published some improvements in the algorithm in 1966. I had the pleasure of meeting him and asked why he did not publish sooner. He said that his field was not numerical analysis and that he was only interested in getting a computer program to do his data analysis. Thus, we see another failure in communication and lost opportunities, the primary point of Dick Garwin's 1969 Arden House keynote address.

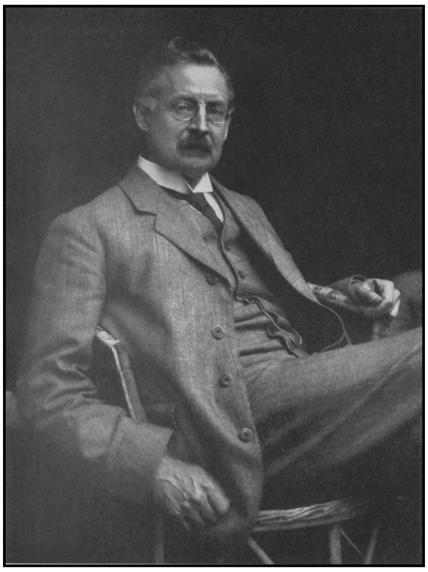
Before continuing further with the discussion of the old literature on the FFT, I would like to point out two important concepts in numerical algorithms which had been stated long ago but did not have very much impact until they were demonstrated by the implementation of the FFT on electronic computers. The first is the divide-and-conquer approach. If a large N-size problem requires effort that increases like W, then it pays to break the problem into smaller pieces of the same structure. The second important concept is the asymptotic behavior of the number of operations. Obviously this was not significant for small N and, by habit of thought, people failed to see the importance of early forms of the FFT algorithms even where they would have been very useful.

I can illustrate this point by going back to the Danielson and Lanczos paper. They describe the numerical problem of computing Fourier coefficients from a set of equally-spaced samples of a continuous function. It is not only a long laborious calculation, but one is also faced with the problem of verifying accuracy. Errors can arise from mistakes in computing or from undersampling the data. Lanczos pointed out that although his use of the symmetries of the trigonometric functions, as described by Runge, reduced computation by a significant factor, one still had an NL algorithm. In a previous reading of this paper, I obtained and published the mistaken notion that Lanczos got the doubling idea from Runge. In fact, he only attributes the use of symmetries to Runge, citing papers published in I903 and in 1905 which l could not find. The Stumpff paper gave a refererence to Runge and Konig. which does contain the doubling algorithm and which appears to have been a standard textbook in numerical analysis. Thus, it appears that Lanczos indepcndcntly discovered the clever doubling algorithm and used it to solve the problems of computational economy and error control. He says, in the introduction to on page 366, "We shall show that, by a certain transformation process, it is possible to double the number of ordinates with only slightly more than double the labor." He goes on to say :

*"In the technique of numerical analysis the following improvements suggested by Lanczos were used: (1) a simple matrix scheme for any even number of ordinates can be used in place of available standard forms; (2) a transposition of odd ordinates into even ordinates reduces an analysis for 2n coefficients to two analyses for n coefficients; (3) by using intermediate ordinates it is possible to estimate, before calculating any coefficients, the probable accuracy of the analysis; (4) any intermediate value of the Fourier integral can be determined from the calculated coefficients by interpolation. The first two improvements reduce the time spent in calculation and the probability of making er-rors, the third tests the accuracy of the analysis, and the fourth improvement allows the transform curve to be constructed with arbitrary exactness. Adopting these improvements the approximation times for Fourier analyses are: 10 minutes for 8 coefficients, 25 minutes for 16 coefficients, 60 minutes for 32 coefficients, and 140 minutes for 64 coefficients."*



*Prof. John W Tukey photo credits alchetron.com*



*Prof. Carl Runge photo credits wikimedia.org*

# MATH IN MEDIA

## Movie Review "Proof"


*Proof movie poster* *a still from movie - proof* *photo credits rottentomatoes*

*This review is based on the movie Proof which is in turn based on a play of the same name by David Auburn. The play won a Pulitzer Prize and a Tony Award, and was performed on Broadway for 917 show. The movie does a good job of portraying the complex relationship between Catherine and her father.*

David Auburn's brilliant play Proof, which opened on Broadway in 2000, is now a Miramax film starring Anthony Hopkins and Gwyneth Paltrow as mathematicians, father and daughter. In recent years, there have been several feature films involving mathematicians, and Proof is one of the best. Hopkins plays Robert, a man who at one time was a brilliant young mathematician but is beset by mental illness. The diagnosis is not specified, but one can infer that it is schizophrenia. Paltrow plays Catherine, who seems to have inherited her father's brilliance as well as his instability.

Here there are parallels like comparing the character of Robert in the movie Proof to the real-life mathematician John Nash Jr. Both men were brilliant mathematicians who suffered from mental illness. Robert, like Nash, believed that there were encrypted messages being communicated to him through newspapers and magazines. He also attempted to do mathematics during his illness, but his efforts were unsuccessful.

Like Nash, Robert suffers from an illness that causes him to believe that there are encrypted messages being communicated to him via newspapers and magazines. Like Nash, Robert attempts to do mathematics during his illness but his efforts reveal not brilliance but the tragic depths of his mental illness. Robert is said to have done revolutionary work in three areas before the age of thirty: game theory, algebraic geometry, and nonlinear operator theory. That seems close enough to Nash that the reference there is impossible to miss. But Proof is more about the daughter Catherine than it is about the father Robert. Auburn must have been fascinated by the story from Nasar's book of John Nash III, who like his father suffered from mental illness. The setting for Proofis Catherine's struggle as the offspring of a famous, brilliant, but unstable mathematician father. She has extraordinary mathematical ability, allowing her to tackle a research problem for which her limited formal education ought not have prepared her. In this respect, she is something like the title character in the film Good Will Hunting. Both Will and Catherine are unusually brilliant but need psychiatric help, he for childhood trauma, she for depression (or is it schizophrenia, like her father?). Both resist treatment. Will gets his and, predictably, weeps and is cured. Catherine, on the other hand, refuses to submit to the psychiatric plans that have been arranged for her. Still, her need for psychiatric help is apparent.

The movie Proof is unusual in that the main character, Catherine, is a woman. When Catherine asks Hal, a young mathematician, if he knows any female mathematicians, he is unable to name any. He eventually remembers Sophie Germain, a famous French mathematician who lived in the 18th century. This shows that Hal is not familiar with the work of contemporary female mathematicians.

Of these three films, Proof is the one that most realistically illustrates the world of mathematics and mathematicians. Matt Damon and Ben Affleck co-wrote and co-star in Good Will Hunting. It is clear that they are fascinated by the story of Srinivasa Ramanujan. Their brilliant yet unschooled character Will, like Ramanujan, emerges from the wrong side of the tracks and clashes cultural with the mathematicians with whom he collaborates. Perhaps Good Will Hunting was envisioned doing for the legend of Ramanujan what West Side story did for the legend of Romeo and Juliet. It is a kind of fictionalization of the Ramanujan story set among modern street toughs. But it is not clear that Damon and Affleck have much of an understanding of how and why mathematicians do what they do. The combinatorial problems on the black board that Will solves are not the type to baffle professional mathematicians. And there are phrases that ring hollow to a mathematician's ear. For example, the fictional Fields medalist from MIT describes Ramanujan as having created "some of the most exciting math theory ever done". A mathematician wouldn't use that phrasing. This Fields medalist also uses the phrases "solve a theorem" and "prove a problem". These mix-ups broke the spell for me. The feeling of actual mathematicians doing actual mathematics is not present in this film.

*a still from movie - proof* *photo credit - pluggedin.com*

The movie Proof is more realistic in its portrayal of the mathematical life than other movies about mathematicians. The three main characters in the film (Robert, Catherine, and Hal) are all mathematicians, and we see them reading, studying, and writing mathematics. Robert describes the pleasure he feels when mathematical ideas are flowing, and Catherine describes to Hal how she felt when doing mathematics, speaking of "beautiful, elegant proofs, like music". Hal describes his fear that his own mathematics research doesn't pass muster when compared with Robert's. David Auburn did his homework and is really able to convey how mathematicians work. The script reveals that Auburn knows about many famous mathematicians, including John Nash, Srinivasa Ramanujan, Sophie Germain, Paul Erdos, and Andrew

Wiles. Auburn's characters seem very much like they belong in the world of real mathematicians.

*Proof* is not just about mathematics. It is also about the human emotions that are involved in the pursuit of mathematics. The film explores the themes of love, loss, madness, and genius. It is a complex and thought-provoking film that does a good job of portraying the life of a mathematician.

There are a couple of breaks from realism in Proof where characters speak in a way that is for the benefit of the audience rather than the way mathematicians would actually talk among themselves. When Hal remembers what a Germain prime is, he speaks to Catherine in a way that would be patronizing to another mathematician. After giving the definition, he offers: "Like two. Double plus one is five, another prime." When Robert and Catherine recall the fact, made famous by a story of Ramanujan and G. H. Hardy, about the number 1729 - the smallest integer that can be expressed as the sum of two cubes in two different ways - they similarly explain too much to each other. Mathematicians, particularly father and daughter, would have a silent rapport on this Auburn and Rebecca Miller, daughter of playwright Arthur Miller, wrote the screenplay for the film, which was directed by John Madden (of Shakespeare in Love fame). Madden should be credited with capturing the feeling of the mathematical world; he consulted effectively with Fields medalist Timothy Gowers of Cambridge University in preparation for the film. It is richer and deeper, simultaneously both funnier and more serious, than either A Beautiful Mind or Good Will Hunting. David Auburn has more to say to mathematicians than do Damon, Affleck, or Akiva Goldsman, the screenwriter for A Beautiful Mind. Proof is a multifaceted story about sibling rivalry, about gender ability (did Lawrence Summers catch it?), about mental illness, about trust in relationships, all set within the world of mathematics. It is also a mystery about the authorship of a discovered manuscript.

Great stage plays rarely make blockbuster movies, and this may be another counter example. The play running for an astonishing 917 performances on Broadway is no mean achievement. And that too for a first effort from Auburn! But one feels, Proof works slightly better on the live stage, with its stark setting and small cast, and with intermission to punctuate the shocking last line of the first act. In particular, part of the fascination of the play is the way the action revolves neatly around the one setting, the back porch of Robert's and Catherine's home. The activities in other parts of the house and other parts of town are implied, cleverly woven into the action back on the porch. One of my favorite lines from the play is absent from the film: When Catherine asks what it means about her mental health that she is having a drink with (an image of) her deceased father, her father replies sadly and poignantly "It could be a bad sign." The film reveals the last name of Robert and Catherine: Llewellyn. There are far more flashbacks in the film, some even momentary. Catherine is 27 and 24 years old in the film where she was 25 and 21 years old in the play. This is presumably to accommodate Paltrow, who was 32 years old when Proof was filmed.

However film medium adds a new dimension to any stage play. A camera can follow a character in a way that an audience cannot. The film Proof gets around Robert's and Catherine's house. So we have the requisite bedroom scene in the film, a scene that is delicately left to our imagination when we see the play. We visit the party that is merely heard offstage in the play. The film gets out into Chicago, Auburn's home town, as well. There are crowds of people at a funeral and on the campuses of the University of Chicago and Northwestern University. The play, by contrast, has but four characters in it. Jake Gyllenhaal, who plays Hal in the film, gives a warm and sympathetic portrayal of a young mathematician, more charming than Hals I've seen on stage.

And *Hope Davis* gives a fine performance in the film as the sane, normal, but ultimately annoying Claire, sister to Catherine. The film adds a couple of minor characters, university mathematicians, with spokenlines. Still, on the whole the movie is similar to the play. To say "based on the play by David Auburn" is to understate the connection. Entire scenes are taken verbatim from the play. Roughly 80 percent of the lines in the film are straight from the play. By contrast, the film version of "A Beautiful Mind" is only loosely based on Nasar's book. The real significance of the film is that it brings the story to a wider audience, just as A Beautiful Mind was seen in film version much more than it was read in book version. (Nasar's book was a New York Times best seller, but far fewer Americans read books than see movies.) The vast majority of Americans may well have an image of a mathematical genius that is shaped primarily by the case studies of Will Hunting, John Nash, and Robert and Catherine Llewellyn in Proof. I suspect that they will draw the following conclusions about mathematicians from these feature films: One identifies five stereotypes about mathematicians that are perpetuated in the movies Proof and Good Will Hunting. These stereotypes are:

**I - Mathematicians are disturbed and need psychiatry**

Since we see that Will is emotionally disturbed, John is paranoid and schizophrenic, and Catherine suffers from depression (at least). A reasonable inference is that mathematical talent is itself a psychiatric illness, that madness is a natural result of a mind that can reason mathematically. Or perhaps it is the converse, that madness induces a state in which the ability to reason mathematically is heightened. It is never easy to infer causation from correlation.

**II - Mathematicians are arrogant and rude.**

Will carries his intellect like a weapon, brandishing it on psychiatrists, on irritating Harvard students, and even on the Fields medalist to demonstrate his superiority. John is portrayed as obnoxious, such as when he cuts down a colleague by telling him that his ideas have not an ounce of originality in them. Catherine, too, seems rarely to be nice to anyone but her father.

**III - Mathematicians are antisocial.**

Neither John nor Catherine seems to have any friends. Will does have friends, but his behavior lands him repeatedly in jail. Hal actually describes mathematicians as wild party animals, but that characterization seems to be mostly for laughs, since the stereotype is opposite.

**IV - Mathematicians are competitive and self promoting.**

They are more interested in advancing themselves, in being recognized as brilliant, than they are in advancing mathematics. Will's only interest in mathematics seems to be as a tool to demonstrate intellect. The young John searches mightily for some big idea that will make others notice him. Catherine accuses Hal of stealing mathematical results for his own advancement.

**V - Mathematicians are young**

Will certainly is young, while the aging Fields medalist seems uncertain whether he can understand Will's work. John does all his work when young, certainly. And Hal worries that he too is over the mathematical hill at age twenty-six. Once mathematicians reach a certain age, Hal in the play suggests that they "might as well teach high school". In the film, Hal quips? I'm twenty-six. You know, the downward slope.? About the assumption that mathematical ability is the province of the young, Robert in the play says "this is a stereotype that happens to be true."

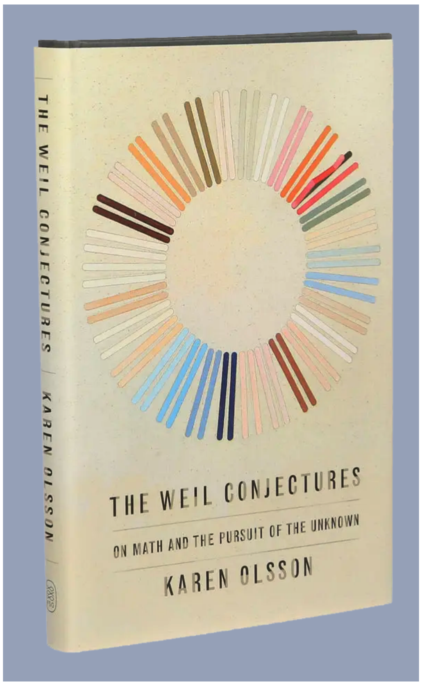
One can note that these stereotypes are harmful because they can discourage people from pursuing mathematics. He also points out that these stereotypes are not always accurate. For example, not all mathematicians are disturbed or arrogant. And while some mathematicians may be competitive, they are also often collaborative and supportive of each other.

*a still from movie - proof* *photo credits - alchetron.com*

Regarding arrogance, it is common experience that mathematicians tend to find mathematics humbling and that they share with other mathematicians a certain fellowship. It is quite suspect that mathematicians may be less social on average than the general population, but probably the same can be said for others who work in cerebral disciplines, where extended solitary concentration is required. I doubt self-promotion is a trait that is attributable to mathematicians more than to any other professionals.

And while many young mathematicians accomplish great things, I doubtvery much that mathematical ability must necessarily wane with age. Much more likely, it is mental energy that wanes with age. These films associate mathematicians with brilliance rather than diligence, which in turn suggests that mathematical work is easy rather than hard. It makes sense Although Robert is mentally ill and Catherine is antisocial, Proof does as much to dispel stereotypes as it does to reinforce them. It presents mathematicians who seem passionate about their work for its own sake, and in this respect it presents a more realistic picture of the mathematicians I know. Hal is musical, athletic, energetic, and funny, but also vulnerable, in many ways exactly like a regular person who is a mathematician. *Proof* shows us mathematicians, young and old, working together with fervor to examine an exciting new manuscript. One imagines it might be a proof of the Riemann hypothesis or perhaps the twin primes conjecture. Perhaps in the sequel we'll find out which result it is and get the details of the proof. Then that a failure to be productive is seen as a loss of intellect rather than a loss of drive or stamina.

# BOOK REVIEW




*The Weil Conjectures: On Math and the Pursuit of the Unknown*

*- by Karen Olsson*

The book in review is a general book aimed at an average student of mathematics who likes to draw inspiration from celebrity mathematicians. The background to the narrative is as follows: In 1940, Simone Weil urged her brother André to write an expository account of his mathematical research, but André replied that explaining mathematical research to non-specialists is like explaining a symphony to a deaf person. He believed that it could be attempted, but in the end, it would result in a kind of poem, unrelated to the thing it pretends to describe.

André's response to Simone's request to write an expository account of his mathematical research is seen as surly and arrogant, but there is truth in it. The latest ideas from the frontiers of research are often not fit for armchair consumption, and it is the nature of a frontier that one has to do some bushwhacking to get there. However, if the discoveries of a research community are so abstruse that they can never be understood outside a small coterie of initiates, then what is the point of discovering them? Somehow, the explorers of new territory have to send the occasional dispatch back to civilization, to let us know what they've found.

Simone Weil asked her brother André to write something expository, but the highly technical treatise he wrote was difficult for her to understand. Simone continued to worry that mathematics was becoming too remote from ordinary life and wanted her brother's work to serve human needs or at least reveal something about the world we all live in.

Karen Olsson's book **"The Weil Conjectures: On Math and the Pursuit of the Unknown"** describes mathematics as a powerful intoxicant, a door to euphoria, and tells the story of the Weil siblings, André and Simone. Olsson writes about her own love of math and shares Simone's concerns about mathematics becoming too remote from ordinary life. Olsson believes that mathematics should speak for itself and wants to understand it in mechanistic detail. The challenge in explaining mathematics is not just to translate the vocabulary into plain language, but to also convey why the mathematical idea is worth understanding. Even a relatively simple theorem like Fermat's Little Theorem can be difficult to explain in ordinary language, and it is hard to explain why it is interesting without using more math.

However, Olsson seems to be baffled by Simone's writing and insensible to her charisma (Despite her challenges, Simone went on to have a distinguished career as a philosopher, political activist, and mystic. She wrote extensively on topics such as labor, poverty, and the nature of God. Her work has been praised for its originality, rigor, and compassion). Olsson explores the work of both Weils, hoping to understand their shared commitment to truth. Simone used math as a meditative practice, allowing it to "surpass the part of the brain that does math." The author seems to understand both sides of the debate about the role of mathematics in society. In fact we explain further that the author tries to weave her own story to justify her arguments.

On the one hand, she agrees with André Weil that mathematics should be understood in its own terms, without resorting to metaphors or analogies. On the other hand, she also shares Simone Weil's concerns about the relevance of mathematics to everyday life. She asks herself, "Why should I care about these forbidding abstractions? What do they have to do with my life as a writer, a parent, a citizen?" She doesn't have any clear answers, but she is still drawn to mathematics, as if echoing David Hilbert's famous statement, "We must know! We will know!"

Before I go any further, I want to make it clear that this book is not a textbook or a scholarly work. It is not written for mathematicians. However, it does raise some questions about the relationship between mathematics and society that may be of interest to mathematicians. This issue is often discussed in terms of outreach, which is the challenge of communicating advanced mathematics to the public. In Olsson's case, it also becomes a question of inreach: how can we help someone who is drawn to mathematical ideas but does not have the necessary background knowledge?

The main part of Olsson's book is a personal essay about her own experiences with mathematics. She describes her own intense and difficult journey into the world of mathematics. Her story is intertwined with the stories of the Weil siblings, who also had intense and difficult lives. André Weil was a child prodigy who grew up to be a rebellious and arrogant mathematician. He was one of the founders of the Bourbaki group, a group of mathematicians who revolutionized the way mathematics was taught and studied. While imprisoned in France during World War II, he proved a variant of the Riemann hypothesis for curves over finite fields. The conjectures mentioned in Olsson's title extend this result from curves to varieties, which are higher-dimensional versions of curves. These conjectures also forge an unexpected link between two seemingly unrelated fields of mathematics: number theory and topology. The conjectures have since been proven by Alexander Grothendieck and Pierre Deligne, but they are still known as the Weil conjectures.

Simone Weil was considered a child prodigy, and her family doted on her. She was yanked in and out of the finest schools by her parents So she was just as brilliant and precocious as her older brother, André. She was his first student and they were inseparable as children. They would memorize long passages of poetry and recite them to each other, slapping each other on the face if they made a mistake. Both Simone and André went on to study at the École Normale Supérieure, but Simone eventually switched from mathematics to philosophy. She then became involved in social and political activism, and later in mystical theology. She was unable to live a comfortable life while others were suffering, so she worked in factories and mines, even though she was not suited for these jobs. She volunteered on the Republican side in the Spanish Civil War, but her time at the front ended when she stepped into a cooking pot and scalded her leg.

André Weil was reluctant to join the military during World War II, not because he was a pacifist, but because he felt that his calling was to do mathematics, not fight in a war. He was eventually arrested and imprisoned for refusing to serve, but he was released after a few weeks when he agreed to report for military service. However, the French forces surrendered a few weeks later. The Weil family escaped Europe shortly before the Nazis closed the borders. In the United States, André had several years of wandering before he found a permanent position at the University of Chicago. He later moved to the Institute for Advanced Study. In 1942, Simone Weil insisted on returning to Europe with the intention of parachuting into battle and nursing the wounded. However, she died before she could fulfill this fantasy, succumbing to tuberculosis and self-imposed malnutrition at the age of 34.

The Weil siblings were fascinating and accomplished people, but their stories have been told elsewhere. In this review, I want to focus on the more personal part of Olsson's narrative. As a young woman, Olsson was drawn to the writings and life story of Simone Weil, but she also developed an early love of mathematics. This eventually led her to an interest in André's work. Olsson writes that she was one of those "young kids" for whom the word "algebra" had a magical shimmer. She couldn't wait to learn more about mathematics, and she found that the pleasure of thinking hard about math was like the mental equivalent of having gone for a long run. In her sophomore year at Harvard, Olsson chose to major in mathematics. She remembers late-night walks across the snowy campus, feeling a kind of euphoria that came only after having thought hard about math.

Coming back to the author Karen Olsson, she describes her own experience of being a math major at Harvard as a "small band of students giddily, exhaustedly trekking through an abstract moonscape." She describes the feeling of being surrounded by people who were just as passionate about math as she was, and the sense of camaraderie that came from working together to solve problems. She also writes about the relationships that she formed with her fellow math students, including her first serious boyfriend. She says that part of what made her love math was the fact that she could share her passion with someone else who understood it. Olsson's story is a reminder that math can be a social and collaborative activity, as well as a solitary one. It is also a reminder that the beauty of math is not just in the solutions, but also in the journey.

In this sense Olsson's book is part memoir, part biography, and part a general history of 20th-century math, and its method is aphoristic and digressive. Continuing her own story she says Although she loved mathematics, her real ambition was to be a novelist. After graduating from Harvard, she put her math degree aside and became a newspaper reporter and editor in Austin, Texas. She eventually published a couple of novels, both of which were set in a political milieu. It wasn't until 20 years later that her love of mathematics came back. This was partly due to her young son's interest in the subject. He would often ask her to give him algebra problems to solve. But later on she turns to youtube for help and then corresponds to a friend who is an academic, Here is a nice description. In the middle of watching the twenty-first online algebra lecture, I hit a wall .... While Professor Gross was elaborating on the Sylow theorems, as he was saying that "any two p-Sylow subgroups H and H' are conjugate," I became instantly tetchy, I could not take it any longer. Who cares? I am a midlife mother of two, I thought morosely, and this is the most pointless thing I could possibly be doing"

In the neighborhood supermarket, she spots the professor who never answered her email, and chases him through the aisles until she corners him with her shopping cart in the tortilla section. He apologizes for not responding. They speak about getting together some time to talk math, but they don't set a date.

As Olsson recounts the setbacks in her own mathematical journey, she also tells the story of André Weil's later years. She shares some of his daughter Sylvie's stories about him, and she admires an elegy written by Goro Shimura. Olsson wonders if she is writing an elegy for mathematics or for her own entanglement with it. She says that at times it feels that way, but she doesn't think that's what she is doing. She says that even though she doesn't need to remember or know ring theory, she still wants to finish the algebra lectures. Olsson's story is a reminder that the journey of learning mathematics can be both challenging and rewarding. It is also a reminder that even when things get tough, it is important to keep going.

One enjoys reading Olsson's book, but it is a disappointed with the way it ended. We hoped to read her account of what the Weil conjectures are all about and what they mean to her, even if it was a brief and incomplete explanation. The initial part is impressive with Olsson's talent for mathematical exposition, as evidenced by her clear and concise explanation of a fixed point of a continuous function. But the story about her getting equipped with algebra with "YouTube" and other resources may bring some remorse to the average reader who may agree with the kind of feelings Olsson describes.

Although Olsson's story can be exasperating at times, on the whole we find her quest inspiring. It's not every day you meet a journalist and novelist who longs for a deeper understanding of varieties over finite fields and their zeta functions. She is not doing it for grades or for glory, but simply because something about mathematics calls out to her. I hope she will continue, and eventually find fulfillment rather than frustration.

Overall for those from the professional mathematics side Olsson's experience raises the question of how research-level mathematics can be made comprehensive to those outside the field. Is it possible? Is it worth the effort? André Weil seemed quite comfortable with the idea of mathematics as an elite guild, open only to those of exceptional talent. He believed that the rest of the world is deaf to the symphony of mathematics. Perhaps he was right, but if so, the situation is rather sad. Beautiful mathematics is played in an empty concert hall, with no one but the composer and the orchestra able to appreciate it. And there's a practical concern: in general, it's the audience that provides material support to the musicians. However, Weil's inward-looking view is certainly not universal. For many others, mathematics is something worth sharing. It is a thing of beauty, a useful tool for understanding the world we live in, and a window onto an unexpected universe. These people work to engage the public through teaching, lectures, expository writing, and mentoring.

Olsson's story offers a bit of cheerful news to these *"evangelists":* it's proof that someone out there is listening, keen to hear the message. But it also underlines how much hard work is needed to open a line of communication between research mathematicians and the general public. So it is indeed possible and worthwhile to make research-level mathematics intelligible to those outside the field. However, it is a challenging task that requires a lot of hard work and dedication.

# CLASSICS

## From Fourier Analysis to group representations and Data Analysis

**Introduction**

There is this famous paper by L. Auslander and R. Tolimieri entitled "Is computing with the finite Fourier transform pure or applied mathematics?" appearing in the Bulletin of AMS, New series, 1979. What follows is a sort of modern commentary on the same. This kind of a discussion has appeared at several places, in journals of mathematics and allied areas, to understand the perceived differences between the so called applied and pure mathematics.

Nowadays we see many engineers and technocrats referring to the term FFT's and DFT's. To start off, The Fourier transform converts an image from the spatial domain to the frequency domain, where each point represents a particular frequency contained in the original image. To elaborate a bit more the Fourier transform (FT) is a mathematical operation that decomposes a signal into its constituent frequencies. In medical imaging, the FT is used for many applications, such as filtering images to remove noise, reconstructing images from incomplete data, and analyzing images to identify features of interest.The output of the FT is a representation of the image in the frequency domain. The frequency domain is a representation of the image in terms of its frequencies. Each point in the frequency domain represents a particular frequency contained in the spatial domain image.

Auslander referred to in the introductory remarks was trained as an applied mathematician at a time when there was no clear distinction between pure and applied mathematics. Mathematicians were free to pursue their interests, whether they were theoretical or applied. However, over time, a divide began to develop between the two disciplines. Pure mathematicians became more focused on abstract problems, while applied mathematicians became more focused on solving problems in the real world. Auslander believed that the two disciplines were not as separate as they seemed. He argued that pure mathematics could provide valuable insights into applied problems, and vice versa. In his paper with Tolimieri, he hoped to show the ultimate unity of pure and applied mathematics.

It looks like there are these detrimental effects of the divide between pure and applied mathematics. On the one hand, applied mathematicians had fewer tools to bring to problems. This is probably because the applied scientists are not exposed to the same level of theoretical training as pure mathematicians. On the other hand, pure mathematicians seem to often ignore the fertile bed of inspiration provided by real world problems. Auslander hoped that his paper would help to *mend* the rift in the mathematical community. He believed that by showing the ultimate unity of pure and applied mathematics, he could encourage mathematicians to work together again.

**The Cooley Tukey Algorithm**

*Now we shall describe the Cooley-Tukey algorithm and the group representations associated. Just as Auslander and Tolimieri concentrated on relations to nilpotent harmonic analysis and theta functions, we emphasize connections between the famous Cooley-Tukey FFT and group representation theory.*

The DFT has several representation theoretic interpretations. One interpretation is that it is the exact computation of the Fourier coefficients of a function on the cyclic group $\mathbb{Z}/n\mathbb{Z}$. This means that it can be used to find the frequencies that are present in a signal.[to be edited for correctness] Formally one can view DFT as a linear transformation that maps a complex vector of length n to its fourier transform.

$$i.ef = (f(0), f(1).....f(n-1))^t$$

belonging to $C^n$ is mapped to its fourier transform $\hat{f}$ belonging to $C^n$.

We know that the k$^{th}$ component of the DFT of f at frequency k is given by

$$\check{f}_k = \sum f(j)e^{(2\pi ijk/n)}$$

where i = $\sqrt{-1}$ and one can similarly workout the inverse of fourier transform.

If one observes its linear algebra closely, computing DFT directly requires n$^2$ scalar operations, whereas the FFT is a family of algorithms that does the job in O(n log n) operations. A similar reduction in complexity is seen for inverse fourier transform as well. Indeed, the DFT diagonalizes any group invariant operator, making possible the following algorithm:

step(1) compute the Fourier transform (DFT).

step(2)Multiply the DFT by the eigenvalues of the operator, which are also found using the Fourier transform.

step (3) Compute the inverse Fourier transform of the result.

This technique is the basis of digital filtering and incidentally it is also used for the efficient numerical solution of partial differential equations.

Though not in the language of Fourier theory, the computations also occur in earlier works of Gauss, Legendre, Dedekind and Frobenius [1]

In fact Frobenius shows that a certain linear form (given below) is a “generic” DFT applied at the frequency $\chi$.

$$\Theta_D(G) = \prod_{\chi \in \hat{G}} \left( \sum \chi(g) x_g \right)$$

where $\chi$ is a character for G i.e a 1-dimensional and representation of G.

In group theory, a character of a group is a function that assigns a complex number to each element of the group. The irreducible characters are a special set of characters that are linearly independent and have certain other properties.

The Fourier transform can be interpreted as a change of basis in the space of complex functions on a group. This means that it can be used to represent a function in terms of its irreducible characters. Consider $\mathbb{Z}$n=$\mathbb{Z}$/n$\mathbb{Z}$, The irreducible characters of $\mathbb{Z}$/n$\mathbb{Z}$ are given as follows:

Given a complex function f on $\mathbb{Z}$/n$\mathbb{Z}$ we may expand f in the bases of irreducible charcter $\{\chi_k\}$ defined by $\{\chi_k\}(j) = e^{2\pi ijk/n}$. In fact the coefficient of $\chi_k$ in the expansion is equal to the scaled Fourier Coefficient $\frac{1}{n}\hat{f}(-k)$, whereas the Fourier co-efficient $\hat{f}(k)$ is the inner product of the function values of $f$ with that of the character $\chi_k$.

The reason for the scaling factor of 1/n is that the characters $\chi_k$ are normalized so that their inner product with themselves is equal to 1.

For an arbitrary finite group G there is an analogous definition. The characters $\chi(n)$ of $\mathbb{Z}$/n$\mathbb{Z}$ are the simplest example of a matrix representation, which for any group G is a matrix-valued function $\rho$(g) on G such that $\rho$(ab) = $\rho$(a)$\rho$(b), and $\rho$(e) is the identity matrix. Given a matrix representation $\rho$ of dimension d$_\rho$ , and a complex function $f$ on G, the Fourier transform of $f$ at $\rho$ is defined as the matrix sum

$$\hat{f}(\rho) = \sum_{x \in G} f(x).\rho(x) - - - - - (Eq1)$$

$$(Eq1)\ can\ be\ expanded\ as\ \hat{f}(\rho_{ij}) = \sum_{x \in G} f(x)\rho_{ij}(x)$$

thus requiring one to compute $d_\rho^2$ scalar fourier transforms.

A set of matrix representations R of G is called a complete set of irreducible representations if and only if the collection of matrix elements of the representations, relative to an arbitrary choice of basis for each matrix representation in the set, forms a basis for the space of complex functions on G. The Fourier transform of f with respect to R is then defined as the collection of individual transforms, while the Fourier transform on G means any Fourier transform computed with respect to some complete set of irreducibles. One can explicitly give the expression for inverse Fourier transform also.

Thus we get a relation between the group Fourier transform and the expansion of a function in the basis of matrix elements. This shows us a relation between the group Fourier transform and the expansion of a function in the basis of matrix elements.
Thus computational theory meets linear algebra and more specifically group representations.
A fair amount of attention has been devoted to developing efficient Fourier transform algorithms for the symmetric group. One motivation for developing these algorithms is the goal of analyzing data on the symmetric group using a spectral approach. In the simpler case of time series data on the cyclic group, this approach amounts to projecting the data vector onto the basis of complex exponentials.
The spectral approach to data analysis makes sense for a function defined on any kind of group, and such a general formulation is due to Diaconis.
The Fourier transform can be a powerful tool for identifying coalition effects in data analysis. It can be used to identify which restaurants are often ranked together and to quantify the strength of these coalition effects. In this example, a group of people are asked to rank a list of 4 restaurants in order of preference. This can be represented as a function on the symmetric group $S_4$, which is the group of all permutations of 4 objects. The Fourier transform can be used to identify coalition effects by looking at the coefficients of the Fourier transform. The coefficients at the matrix elements $\rho_{ij}(\pi)$ of the (reducible) defining representation count the number of people ranking restaurant i in position j. This means that the Fourier transform at $\rho_{12}(\pi)$ tells us how many people ranked restaurant 1 in the first position and restaurant 2 in the second position.

Similarly, the Fourier transform at $\rho_{ij}(\pi)$ of the (reducible) permutation representation of Sn on unordered pairs $\{i, j\}$ counts the number of respondents ranking restaurants i and j in positions k and l. This means that the Fourier transform at $\rho_{12}(\pi)$ tells us how many people ranked restaurants 1 and 2 in the same position, regardless of whether it is the first position or the second position.

Here are some other applications of the Fourier transform to data analysis:

- Identifying patterns in time series data
- Detecting anomalies in data
- Compressing data
- Filtering noise from data
- Analyzing images
- Analyzing audio .

**References**

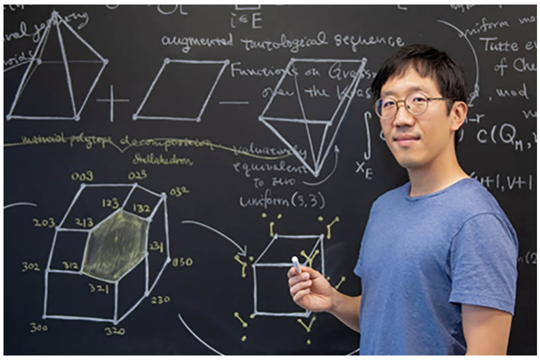
[1] David K. Maslen, Daniel N. Rockmore, The Cooley-Tukey fft and group theory, modern signal processing msri publications vol 46, 2003.

[2] James W Cooley, How the fft gained acceptance, acm publications newyork 1987.

*mathematical based origami*

## Around The World....The browsing fingers!!

*James Maynard photo credits St John's College, Oxford*




*June Huh photo Credits i.ytimg.com*


*Hugo Duminil photo credits universite-paris*

*Maryna Viazovska photo credits nature.com*

Recently, the NIST, the US standards-setting body, announced the first four algorithms for postquantum cryptography. Such algorithms promise to solve one of the biggest problems created by the advent of quantum computers due to their ability to break public key cryptography used today, for communications on the Internet. Four further algorithms are under consideration for standardization to have greater variety and, therefore, a better overall security level. One of these, SIKE, was unexpectedly broken and therefore cannot become a standard. Not without some irony, the cipher was broken by a traditional computer of limited power.

SIKE, an abbreviation that stands for Supersingular Isogeny Key Encapsulation (encapsulation of keys through supersingular isogenies), a proposed algorithm for standardization designed to replace the current ones, vulnerable to attacks by quantum computers.

The lately announced Shantiswarup bhatnagar award in Mathematical sciences jointly goes to Dr. Apoorva Khare of the Indian Institute of Science and Dr. Neeraj Kayal of Microsoft Research Lab. The work of Khare which bagged the award was abstract algebra, combinatorics and discontinuous structures. His work on matrix analysis and related combinatorial analysis is well known among peers.

Neeraj Kayal on the other hand for his outstanding contributions to Computational Complexity. His extensive, innovative work on algebraic computation includes the development of deep lower bound techniques proving limitations of this natural model, as well as designing efficient algorithms for reconstruction and equivalence of such algebraic circuits.

The 2022 Shaw Prize in Mathematical Sciences to Prof. Noga Alon for the entirety of his groundbreaking work, which has included laying the foundations for streaming algorithms used in Big Data analysis and the development of algebraic and probabilistic methods to deal with problems in combinatorics, graph theory and additive number theory. "He introduced new methods and achieved fundamental results which entirely shaped the field," the judges wrote. Prof. Hrushovski is well known for several fundamental contributions to Model theory with applications to algebraic-arithmetic geometry and number theory, in particular in the branch that has become known as Geometric Model theory. His Ph.D. thesis revolutionized Stable Model theory (a part of Model theory arising from the Stability theory introduced by Saharon Shelah). Shortly afterwards he found counter examples to the Trichotomy Conjecture of Boris Zilber and his method of proof has become well known as Hrushovski construction and found many other. The ICM 2022 announced the four field medals along with other awards.

The 2022 fields medalists are James Maynard, June Huh, Hugo Duminil-Copin and Maryna Viazovska. The last name viz Maryna is only the 2nd woman fields medalist. Her work is in discrete geometry and related analysis. Specifically, she proved that the E8 lattice provides the densest packing of identical spheres in eight dimensions. 35 Year old James Maynard solved one of the discipline's oldest and most central problems, about the spacing of prime numbers. Now a professor at the University of Oxford, was awarded the Fields Medal for his "spectacular contributions in analytic number theory," according to the award citation. "His work is highly ingenious, often leading to surprising breakthroughs on important problems that seemed to be inaccessible by current techniques." Maynard, who was informed of his Fields Medal in January, "wasn't expecting it at all," he said. "I still fundamentally think of myself as someone who's slightly finding their feet in the world of mathematics." He has often drawn inspiration from the work of previous Fields medalists, he said. "Suddenly to be tossed up on this list, with these legends of mathematics who inspired me as [a child], is incredible but completely surreal." According to IMU citation, Hugo Duminil-Copin is awarded the Fields Medal 2022 for solving long-standing problems in the probabilistic theory of phase transitions in statistical physics. His work has led to breakthroughs in the understanding of the Ising model and percolation theory, and has opened up new avenues for research in these areas.

The fourth is June Huh, a mathematician at Princeton University, was awarded the Fields Medal in 2022, and he became the first mathematician of Korean descent to receive the medal. Huh was recognized for his work in combinatorics, specifically for "bringing the ideas of Hodge theory to bear on longstanding problems in combinatorics, including the Rota conjecture and the log-concavity of the characteristic polynomial of a matroid." His work has been described revolutionary and a msterpiece. The citation notes that Huh is awarded the Fields Medal for his work in combinatorics, specifically for "bringing the ideas of Hodge theory to combinatorics, the proof of the Dowling-Wilson conjecture, and the resolution of the log-concavity conjecture for matroids"

Mark Braverman, a professor of computer science at Princeton University, was awarded the 2022 Abacus Medal at the International Congress of Mathematicians (ICM) in Helsinki, Finland. Braverman was recognized for his contributions to theoretical computer science, particularly in the areas of computational complexity theory, algorithmic game theory, and machine learning. He has made significant contributions to the understanding of the computational complexity of fundamental problems in these areas, and his work has had a significant impact on the development of algorithms and machine learning techniques.

The 2021 AMS Award for Impact on the Teaching and Learning of Mathematics has been awarded to Solomon Friedberg of Boston College, where he is the James P. McIntyre Professor of Mathematics.

He is honored for his many significant contributions to mathematics education locally, nationally, and internationally. Working to build a diverse, engaged community. The MSRI-UP program selected for the 2021 Mathematics Programs That Make a Difference Award by the AMS represents a profoundly successful, 14-year-long effort to do exactly that The concept underlying the MSRI-UP program seems simple: invite a group of 18 promising undergraduate math majors from diverse backgrounds - Black, Latinx, women, and other historically underserved populations-to a six-week-long summer research program with the goal of encouraging their interest in mathematics and laying a foundation for them to continue in the field after they graduate. The program is led by a similarly diverse team of dedicated mathematics professors, assisted by graduate students. The 2022 Leroy P steele prize is won by Michel Gomens and David Williamson. In their seminal work, Goemans and Williamson presented a new approximation algorithm for the Max-Cut problem that yields an approximation ratio of 0.878. Richard P. Stanley, an Arts and Sciences Distinguished Professor at the University of Miami, was awarded the 2022 AMS Leroy P. Steele Prize for Lifetime Achievement. Richard Stanley has been a giant in combinatorics and related areas for over four decades. He has revolutionized enumerative combinatorics, revealing deep connections with other branches of mathematics, such as commutative algebra, topology, algebraic geometry, probability, convex geometry, and representation theory. The 2022 Steele Prize for Mathematical Exposition is awarded to Aise Johan de Jong as the originator and maintainer of the online textbook, the Stacks Project. The current year winners are

- Nicholas M. Katz, Princeton University, in 2023 won the Steele prize for Life-time Achievement. The prize is awarded every three years to a mathematician who has made outstanding contributions to mathematics. Katz is a world-renowned mathematician who has made fundamental contributions to number theory, arithmetic geometry, and algebraic geometry. His work has had a major impact on the development of these fields, and he is considered one of the leading mathematicians of his generation.

- The 2023 Leroy P. Steele Prize for Seminal Contribution to Research was awarded to Peter B. Kronheimer and Tomasz S. Mrowka, both of Harvard University, for their paper "Gauge theory for embedded surfaces", It was published in 1993 in the journal Topology.

- Lawrence C. Evans, a professor emeritus at the University of California, Berkeley, has received the 2023 Leroy P. Steele Prize for Mathematical Exposition for his book Partial Differential Equations published by the American Mathematical Society, Providence, RI, 1998 (first edition) and 2010 (second edition). In fact this is his scond steele prize, first he got in 2004 for his seminal work - "the Evans-Krylov theorem"

The 2022 Chevalley Prize in Lie Theory was be awarded to Xuhua He for his substantial advances in at least three directions of Lie Theory: the study of the cocenter of Hecke algebras of p-adic groups, the study of affine Deligne-Lusztig varieties and the theory of modular representations of semisimple groups.

The 2022 Norbert Wiener Prize: For his outstanding contributions to the mathematical theory of wave propagation, Michael Weinstein of the UCLA was awarded the Norbert Wiener prize by the AMS.

India won two gold medals, two silver medals, and two bronze medals at the International Math Olympiad held in Chiba, Japan between July 2 and 13, 2023 India had sent a six-member Indian team won six medals - two gold, two silver, and two bronze at the 64th International Mathematical Olympiad (IMO) The team finished ninth out of the 112 countries that competed. The gold medal was won by Atul Shatavart Nadig and Arjun Gupta, who both scored 37 points.

## Hilbert Correspondences

David Hilbert was a mathematician born in Germany in 1862, who is referred to as the founding father of geometry due to his large contribution to the establishment of mathematics. He attended the University of Konigsberg from 1880 to 1885 and wrote several books, including "Statistical Mechanics," "Theory of Algebraic Number Fields," "The Foundations of Geometry," and "Principles of Mathematical Logic".

Hilbert's work on the finite generation of the algebra of invariants in 1890 resulted in the creation of a new mathematical discipline, abstract algebra.

This work dealt with the question of whether the algebra of invariants for a given group is finitely generated, which means that it can be generated by a finite number of elements. Hilbert proved that this is the case for certain groups, which had important implications for the study of invariants. In a later paper in 1893, Hilbert dealt with the same questions in more constructive and geometric ways, but this work remained virtually unknown until David Mumford brought these ideas back to life in the 1960s in his geometric invariant theory Mumford's work extended Hilbert's ideas to a more general and modern form.

Hilbert's work on invariants had important implications for other areas of mathematics, including projective geometry, algebraic geometry, and algebra . Invariant theory is closely tied to these fields, and its modern resurrection is a significant branch of mathematics. Hilbert's work on invariants also had important implications for physics. The mathematical structures that he developed, including Hilbert spaces, have become fundamental tools in quantum mechanics.

This article is to justify the cover theme that mathematical communications are very important and noteworthy. However no article on Hilbert is complete without the mention of something that was later made popular by Riemann.

David Hilbert was a German mathematician who made significant contributions to geometry, abstract algebra, and mathematical logic. He is best known for his work on the finite generation of the algebra of invariants, which led to the creation of abstract algebra. The work on Invariant theory grew out of his corespondences with Klien. In reality Gordan originally tried his hand on finite generation for certain polynomial rings. But Hilbert wanted to prove it successfully through a simple argument. This is how he was led to his invariant theory.

Hilbert's work on the finite generation of the algebra of invariants dealt with the question of whether the set of all polynomial functions that are invariant under the action of a given group can be generated by a finite number of functions.

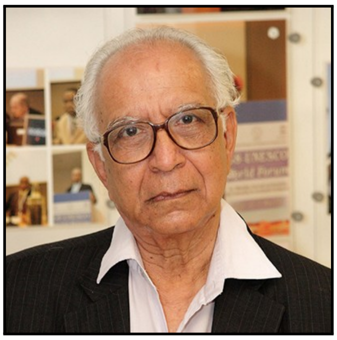
Hilbert proved that this is the case for certain groups, which was a major breakthrough in the study of invariants.

In a later paper, Hilbert dealt with the same questions in more constructive and geometric ways, but this work was not well-known until David Mumford rediscovered it in the 1960s.

Mumford's work extended Hilbert's ideas to a more general and modern form, and it has had a profound impact on the development of algebraic geometry. Hilbert's work on invariants had a big impact on other areas of mathematics, such as projective geometry, algebraic geometry, and algebra.

Invariant theory is closely related to these fields, and its modern revival is an important branch of mathematics. Hilbert's work on invariants also had a big impact on physics. The mathematical structures he developed, including Hilbert spaces, have become essential tools in quantum mechanics.

## Face2Face - Interview with M S Narasimhan



*M. S. Narsimhan*

*Sujatha Ramdorai*

*This is conversation between two mathematician Sujatha Ramdorai from University of British Columbia, Canada Interviewing with M S Narasimhan*

Professor Mudumbai Seshachalu Narasimhan is a highly accomplished Indian mathematician whose seminal work in Algebraic Geometry is recognised worldwide and has made inroads into different areas within mathematics and theoretical physics. He was with the School of Mathematics, Tata Institute of Fundamental Research (TIFR) for a large part of his career and then was the Head of the Mathematics group at the International Centre for Theoretical Physics (ICTP), Trieste from 1992-1999. He currently lives in Bangalore, India. His work has fetched him numerous accolades and prizes, in particular the Shanti Swarup Bhatnagar Prize (1975), the Third World Academy Award for Mathematics (1987), Padma Bhushan (1990), Fellow of the Royal Society and the King Faisal International Prize for Science (2006; jointly with Simon Donaldson, Imperial College).

His eightieth birthday was marked by mathematical conferences in Spain (http://www.icmat.es/congresos/isc2012/) in September 2012, and in Bangalore (http://math.iisc.ernet.in/~imi/ICAG.php) in December 2012. Professor Narasimhan graciously consented to an e-interview with Sujatha Ramdorai in November which was followed-up with a subsequent tête-à-tête in Bangalore.

**Sujatha Ramdorai: At the outset, my warmest greetings on your 80th birthday year. You have had a longand illustrious career as a mathematician. Looking back, what would you say were your best moments that you cherish?**

Mudumbai Seshachalu Narasimhan: The best moments, I think, were the time I spent, as a student, with Fr Racine, K Chandrasekharan and L Schwartz, which shaped my approach to mathematics and my mathematical career.

**SR: Can you tell us a little more about your childhood, the environment at home, your schooling ...** MSN: I come from a small village from Tamil Nadu, (from the now nonexistent North Arcot district) and the nearest secondary school was 5 miles away. I come from a family of agriculturists who were once fairly well off and due to droughts and my father passing away when I was about 12 years old (I was the eldest son), the family was facing reduced circumstances. I was good in my studies, especially in mathematics. I was fascinated by Euclid and the thrill it gave me to solve "riders", thinking for oneself. Even in school I wished to "do research", though I am sure I did not know what it meant. I was encouraged and supported by my family when I wanted to study mathematics and was not under pressure to pursue any other career. When I used to draw mathematical diagrams on the walls of the house, I was presented with a blackboard.

**SR: And your college years?**

MSN: I studied in Loyola College, Madras and it was a great fortune that Fr Racine was teaching in that college. He was in touch with several outstanding French mathematicians. He was attempting to introduce several modern fields of mathematics to Indian students and mathematicians. He was one of the first in India to introduce Modern Algebra at the undergraduate level. Association with him at the formative stage was crucial for my future mathematical development. He instilled in me a taste for good mathematics. It is he who suggested to me to go to the Tata Institute for pursuing research.

**SR: Fr Racine was one of those legendary names one heard in TIFR often. The generation of Mathemati- cians in India who were directly his pupils clearly have warm memories. Could you tell us more about him? Did you stay in touch with him later during your mathematical career? What were his feelings about seeing people under his tutelage do so well in Indian (and international) math?**

MSN: Father Racine was a member of the Society of Jesus and he spent the later part of his life in India teaching mathematics, first in St Joseph's college, Tiruchi and then in Loyola college, Madras. He obtained his doctorate in Paris, working with Elie Cartan. He was in touch with outstanding mathematicians in France, like Leray, H Cartan and Weil and was following mathematical developments in France and tried communicating them to Indian mathematicians. For instance, he lectured on the theory of sheaves in Madras soon after the work of Leray was published and his reviews of this work appeared in Zentralblatt MATH. He had a remarkable capacity for identifying students with talent and aptitude for mathematics and mentoring them. He guided his students in acquiring a broad-based training in mathematics and somehow enabled them to acquire the faculty to discern what is deep in mathematics.

He took particular interest in advising his students in the pursuit of a career in mathematics and also in following their progress. I used to keep in close touch with him and write to him regularly about what I was doing in mathematics. (It gave him and me special pleasure that we could correspond in French during my stay in France). In the later years I got to know him well and he liked to talk about mathematicians and scientists and he had a nice sense of humour. I visited him in a hospital in Bangalore during the last days of his life. Undoubtedly Fr Racine played a major role in the development of mathematics in India. The list of outstanding Indian mathematicians he mentored is impressive. Among his former students were: Minakshisundaram, K G Ramanathan, Seshadri, Raghavan Narasimhan, C P Ramanujam, Ananda Swarup and myself. He was happy and pleased with his role in starting the research career of so many first rate mathematicians and also, I am sure, with their sense of gratitude towards him. I should also mention that he was honoured by the French Government by a Legion d'honneur.

**SR: You were one of the early members of TIFR (Tata Institute of Fundamental Research). Can you reminisce a little about the mathematical scene in the country at that period?**

MSN: At that time (mid 1950s) there was a small number of good mathematicians in India, working in isolation. However, there was no expertise in India in many important fields of modern mathematics. ("Modern" algebra and topology were taught only in one or two universities!) There was no organised support for research nor a proper mechanism for training and channelling the talents of young Indians into creative research in Mathematics. The situation changed after Independence and the Indian Government made available substantial financial resources for the organisation and development of scientific research. The scheme initiated by K Chandrasekharan in TIFR to develop a School of Mathematics at the highest international level, was the turning point for mathematicians and I was one of the early beneficiaries of this development.

**SR: The Narasimhan-Seshadri theorem was one of the results that put TIFR on the international**

**Mathematical map. When did you actually start working on this problem? Can you tell us about how Weil's work eventually led to your work with Seshadri?**

MSN: Already in our student days Seshadri and I were familiar with the paper "Generalisation des fonctions abeliennes" by A Weil. We became aware through K G Ramanathan of this paper which was pointed out to him C L Siegel. By 1960, the theory of vector bundles in topology was well developed and algebraic vector bundles were being studied by Weil, Serre, Grothendieck and Atiyah. It seemed to be an opportune moment to undertake an intensive study of vector bundles on projective varieties. My impression is that we had the problem of vector bundles on curves in our mind from our student days when we became aware of Weil's paper, but started thinking about it seriously in early 1960s when we familiarised ourselves with the theory of deformations of complex structures. From the present day point of view, Weil envisages in this paper a study of holomorphic vector bundles on a compact Riemann surface and attempts to construct their moduli spaces, generalising the construction of the Jacobian (the word "vector bundle" is not found in the paper and Weil works with "matrix divisors" or "adeles" as we will say today). Weil mentions that holomorphic vector bundles arising from unitary representations of the fundamental group of the Riemann surface should play an important role. Seshadri and I first showed that bundles arising from (classes of) irreducible representations of the fundamental group in a fixed unitary group form a complex manifold, using the then emerging theory of Kodaira and Spencer on deformations of complex structures. We realised that the crucial problem was to give an algebraic characterisation of holomorphic bundles arising from unitary representations. We also felt the "Method of continuity" of Klein and Poincaré could help in proving such a result, once the algebraic condition was available. In the Stockholm ICM talk (1962), David Mumford, motivated by Geometric invariant theory, introduced the notion of a stable vector bundle and this turned out to be the sought-after algebraic condition. Seshadri and I proved, in 1964, that a holomorphic vector bundle on a compact Riemann surface arises from an irreducible unitary representation if and only if it is stable and of degree zero. We also proved a corresponding theorem for vector bundles of arbitrary degree by considering unitary representations of suitably defined Fuchsian groups. The proof used a combination of techniques from algebraic geometry, complex analysis, topology and partial differential equations.

**SR: Obviously those early years of TIFR, especially the contacts with the French School shaped the mathematical landscape in India then and into a long future. You were amongst the early mathematical pilgrims, and spent time as a student in France under Laurent Schwartz. Tell us about that and also the mathematical scene in Paris in those years.**

MSN: I was in Paris during 1957 to 1960. During this period Algebraic geometry was being revolutionised by Grothendieck and others; the Cartan and Chevalley seminars were also taking place. There was also much activity in Paris on Partial Differential Equations by the school of Schwartz, streamlining and advancing the subject by a systematic use of the theory of distributions. I was interested at that time in Partial Differential equations, thanks to the course of lectures of Schwartz in TIFR on Complex Analytic Manifolds, especially on Hodge theory. I met in Paris the Japanese mathematician Takeshi Kotake who was visiting Paris to work with Schwartz and we collaborated on a work concerning linear elliptic operators with real analytic coefficients. During this period I studied the huge preprint of Kodaira and Spencer on deformations of complex structures and this turned out to be very fruitful in my future work. Schwartz gave me a copy of this paper. I have the impression that Schwartz himself was thinking on these matters in response to some questions of Weil on deformation of Riemann surfaces (and I guess that Schwartz was one of the “ellipticians” to whom Weil was referring to in his paper on this topic). The work of Kodaira and Spencer used crucially the theory of elliptic PDE with which I was familiar.

The great mathematicians in Paris were easily approachable by young mathematicians. I used to meet Schwartz regularly. I remember that Grothendieck spent some time explaining to me, on my request, his approach to deformation theory.

**SR: Similarly, about the Harder-Narasimhan theorem which is now proving to have unexpected connections and applications to different areas of mathematics and also to Physics.**

MSN: Once the moduli spaces were constructed the problem of computing numerical invariants of these spaces in particular Betti numbers arose. In the case of bundles of rank 2 with fixed determinant of odd degree, the Betti numbers were computed by P E Newstead by purely topological methods, using the description of these spaces in terms of unitary representations. Based on these results G Harder verified (in 1970) the Weil conjectures for this variety (in the case of a curve over finite field) at a time when Weil conjectures were not proved in general. P Deligne proved the Weil Conjecture in 1974. It was then natural to try to generalise the method of Harder and compute the Betti numbers of moduli spaces in the case of vector bundles of arbitrary rank and (coprime) degree, by calculating the number of rational points of the moduli space and using Weil Conjectures. Reinterpreting the fact that the Tamagawa number of SL(n) is 1, “Siegel’s formula” gives an explicit expression (in terms of the zeta function of the curve) for the sum $\sum 1/\#\mathrm{Aut}(E)$, the sum being over all vector bundles E; the sum over stable bundles gives essentially the number of rational points. To compute inductively the sum over unstable part (and hence the number of rational points), one uses a partition of this set by the “type” of the “Canonical filtration” of a vector bundle. Harder and I showed that any vector bundle has a unique filtration by subbundles such that the successive quotients are semi-stable and with their “slopes” (degree/rank) are strictly decreasing and used the type of this filtration, namely the degree and rank of the successive quotients, to partition the space. It turned out that this is a universal principle valid in several situations and enables one to endow an arbitrary object with a canonical filtration whose successive quotients are semi-stable objects.

**SR: You spent almost all your career in TIFR till the age of 60. Can you please share your thoughts and experiences of those years?**

MSN: First, the early days in TIFR, when a whole new world of mathematics was opening up to me as a young student having contacts with outstanding mathematicians, were perhaps the most exciting. I learnt, during my first two years, Functional analysis and Peter?Weyl theorem from Warren Ambrose, Algebraic Topology from Eilenberg (who, without ever mentioning the words “categories” or “functors”, taught the whole course from a functorial viewpoint), and Complex Analytic Manifolds (in particular Kahler manifolds) from Laurent Schwartz, whose course paved the way for many of my future mathematical interests. There were intense discussions and joint seminars with fellow students. I remember a joint seminar with Seshadri on Weyl’s book on Riemann surfaces, which turned out to be important later in our joint work. (There was no English translation of this book at that time.) Conversations with KC and KGR introduced me to various aspects of number theory and arithmetic groups. Bourbaki and Cartan seminars also played important roles in my early mathematical formation. The atmosphere and conditions for creative research in TIFR suited me perfectly. I could pursue my own independent directions of research, without any undue pressure to produce "results" quickly. The broad based interests, training and knowledge acquired in different fields and the excellent library in TIFR were of great help. The mathematician with whom I collaborated (in the fields of Algebraic and Differential Geometries) intensely and over long periods in TIFR was Ramanan. Our way of approach to and thinking about mathematics were very similar. I suppose I never worked so hard as during the period I was working with him.

I have had brilliant students in TIFR, who became eminent mathematicians and who have contributed to the renown of TIFR. I helped to create and develop schools of Algebraic Geometry, Differential Geometry and Lie Groups in TIFR.

During my stay at TIFR I also worked on creating structures and organisations for promoting Mathematical research in India.

**SR: What about the years outside of India after that?**

MSN: I went to the International Centre for Theoretical Physics at Trieste (ICTP) in 1993 as the Head of the mathematics section at the invitation of Abdus Salam. Actually he had invited me a few years earlier but at that time I could not accept the invitation. I was always interested in creating structures for promoting mathematical research in India and in developing Countries. In India as Chairman of the National Board of Higher Mathematical and internationally as member of EC of IMU and President of IMU?s Commission on Development of Exchange, I had some experience in this direction. The aims of ICTP being to "advance scientific expertise in the developing world", working at ICTP provided a very good opportunity for promoting mathematics. I had complete freedom and financial resources at ICTP to set up schemes for this purpose. During my stay at ICTP many young mathematicians from developing countries have used the intellectual atmosphere and facilities at ICTP to establish themselves as leading mathematicians and in turn have built up mathematical research in their countries. After the stay at ICTP, I spent three fruitful years at SISSA (Trieste).

**SR: How would you contrast your experiences of your career within and outside of India?**

MSN: I enjoyed my work and career both in India and abroad; I had ample support from institutions in India and abroad for carrying out my personal research and for working for the development of mathematics. Working abroad at ICTP gave an opportunity to interact with young mathematicians from all over the world and help them in furthering their research. This, like my role in TIFR, gave me immense satisfaction.

**SR: Were there any clear tipping points or turning points in your research career? Any "Eureka" moments?**

MSN: I do not remember any "Eureka" moment, as such. But there were many exciting moments. SR: Talk to us about a few of those.... MSN: There were really many. To name a few: The work with Ramanan on universal connections when things fell into place smoothly and swiftly and when we discovered the relationship between incidence correspondence in projective geometry (related to quadratic complex of lines) and the Hecke correspondence between moduli spaces of vector bundles on curves. Naturally also the work with Seshadri and Harder on stable bundles. A work which gave me much pleasure was the work with K Okamoto on concrete realisation of discrete series representations, especially as, when we started working on the question, I had hardly any experience in this field.

**SR: Some aspects of geometry started out with connections to Physics and Hermann Weyl was**

**one of the early visionaries to perceive these deep connections. What was your perception of these connections when you started out on your research career?**
MSN: Hermann Weyl was a great hero of mine in mathematics; however when I was young I did not appreciate his visionary role in perceiving the deep connections between mathematics and physics. I began to read his writings, both technical and historical, in this area much later. It would be great if there are people like him now who can write about the interaction between mathematics and physics of the present day with profound knowledge of both the fields and with such authority.

**SR: Starting from the 1960s, Algebraic and Differential Geometry forged ahead as Abstract or Pure mathematics. The links with String Theory, etc. were uncovered several decades after the mathematical advances were made. With the work of people like Hitchin, Witten, etc. the connections to Theoretical Physics were brought to the fore again. Today we seem to be in an era where the insights come from Theoretical Physics and the mathematicians are trying to catch up. What is your perspective on these intertwinings?**
MSN: I started looking into some physics literature when I found that physicists were using some of my results with Ramanan and Sephardi by curiosity. I found that physicists had their insights (and discoveries) in certain mathematical problems, these insights apparently coming from some physical intuition. Examples in low dimensional topology, linear systems on moduli spaces and enumerative geometry, coming from gauge theory, conformal field theory, super symmetry come to mind. It seems that at present there are not so many remarkable insights coming from physics as it was a few years back. The major developments in the last few years (e.g. Fermat's theorem, Poincaré conjecture) come from internal dynamics in mathematics.

**SR: Yet, the "internal dynamics" in the advance of these results are interesting. Fermat's theorem built on a vast body of earlier results in mathematics but from other areas, and in turn caused a surge in the area of arithmetic geometry, while Poincaré conjecture was proved using methods from within math, but in unexpected ways. What are your views on these remarkable interconnections within mathematics itself?**
MSN: What fascinated me in mathematics is the exciting, amazing and often unexpected interconnections between various fields of mathematics and how this connection helps one to solve concrete problem in one of the fields. Who would have thought that Fermat's theorem would be related to the problem of modularity of elliptic curves over rationals, and this relationship would be a catalyst to attack the problem of modularity? As for the proof of the Poincaré's conjecture, one can say that it is a triumph of analysis combined with geometry. The deep techniques developed to solve the problem have been useful in solving other outstanding problems, which is a hallmark of a great work.

**SR: What are your views on the state of Higher education and research in India? A SWOT (Strength, Weakness, Opportunities and Threats) analysis.**
MSN: Strength: We have some institutions of top level in undergraduate, graduate and doctoral education. Potentially very talented students. Weakness: Mostly undergraduate education is weak. Many bright students do not want to pursue academic studies leading to original work. Opportunities: Now seem plenty many higher education institutes being started. There are training programmes at various levels and

substantial financial support to students. Threats: Not having many qualified teachers and not too many people pursuing academic career, mainly due to internal and external brain drain.

**SR: Are there any lessons we should be learning from the way things are done in other parts of the world?**

MSN: I do not know. We have availability of resources and generally support for development of mathematics in India. We have some first rate institutions of research and undergraduate training, though small in number for the size of the country. But the "internal and external brain drain" which I mentioned above, is a major constraint in making the Big Leap.

**SR: Besides mathematics, what are your other interests or hobbies?**

MSN: I am interested in literature, both Tamil and English (to a lesser extent French). I read quite a bit of detective fiction. Basically I am addicted to books. I like to listen to music, both carnatic and western.

**SR: After such a long career in different aspects of mathematical research, what words of advice do you have for youngsters, especially those from India, who might want to embark on a research career?**

MSN: I do not know if I have any special insightful advice. First of all get a broad based knowledge when you are a student, by reading good textbooks and seminar notes, classics and masters and by associating with good mathematicians. One often learns faster many branches of mathematics by discussions with teachers and fellow students. At the time of launching into research, one should be in an environment where good mathematics is cultivated, otherwise there is a danger of pursuing trivial research.

When you wish to learn a new subject or wish to pursue a new field of research, try to approach the field from as high and as sophisticated point of view that you are capable of.

**SR: There are several areas of mathematical research where India has no presence. What are your thoughts on establishing a broader research base in mathematics in the country?**

MSN: There are several areas of mathematical research where India has no presence.

**What are your thoughts on establishing a broader research base in mathematics in the country?**

MSN: There are quite a few major areas in India where there is strength (Number theory, Lie groups and arithmetic groups, algebraic and differential geometry, algebra, analysis ...). We should concentrate on strengthening further these fields and increase the number of experts. At the same time, we could identify a few areas where we lack expertise and develop them drawing upon our experience during the past decades in cultivating the areas mentioned above.

SR: Thanks very much, Professor Narasimhan. It has been a pleasure interacting with you, and once again warmest wishes for now and the years ahead.

## Problem corner - By Chidanand Badiger

*Our Problem section is unique in the sense that it provides some aid for early grad students to think about some problems in such a way that along the way they also learn the theory related to the problem or a class of problems. Illuminating solutions shall be featured in subsequent issues.*

**Introduction:** The length and directions are essential to visualise a vector and have a prominent role in Physics and allied branches of sciences. Mathematics gives rigour to both magnitude and directions through the norm and basis of algebraic structure on a non-empty set called vector spaces. The normed linear spaces provide the definition of magnitude for all the vectors in the space. Banach and Hilbert spaces are more generalized in which one can define completeness and angle between two vectors [1-5]. The definition of a norm is important for many reasons and in a normed linear space there exist infinitely many norms [1-5]. It is a question that every linear space are normed linear space. The following is a question for the issue.

**Statement of the Problem:** Let $\mathbb{C}(\mathbb{R}, \mathbb{R})$ denote the set of continuous functions from $\mathbb{R}$ to $\mathbb{R}$ which is a linear space over the field $\mathbb{R}$ then

1. Is there a norm on $\mathbb{C}(\mathbb{R}, \mathbb{R})$? If exists define.
2. Is the norm defined in (1), bounded?
3. With respect to norm defined in (1), is $\mathbb{C}(\mathbb{R}, \mathbb{R})$ a Banach space?
4. Is there an inner product on $\mathbb{C}(\mathbb{R}, \mathbb{R})$? If it exists then define.

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**Note:** Readers are invited for solutions to this problem through email to MSIB (msibelgaum@gmail.com) before 30-01-2021. The accurate solutions that come to us first and which are innovative will be published in the subsequent issue of the MSIB Newsletter including all readers' names who have sent a correct answer. Therefore, it is requested that the reader should send Name, Email, Affiliations, (references if necessary) along with their solution.

**Solution to the last issue problem:**
Show that fixed-point property of an object in category $\mathbb{C} = \mathbf{Top}$ is a Topological invariant.
**Solution:** Let $M$ be an object in $\mathbb{C} = \mathbf{Top}$ then $M$ is a topological space. If $M$ has fixed point property then very continuous functions $f : M \to M$ has a fixed point say $x_0$ i.e. $f(x_0) = x_0$.
Let $N$ be any object in category $\mathbb{C} = \mathbf{Top}$ and $g : M \to N$ be a homeomorphism then $N$ has fixed point property. Because, for every continuous map $h : N \to N$ then $g^{-1}ohog(c) = c$ Therefore, the element $g(c)$ is in $N$ which become a fixed point in $N$ under $h$ because $h(d) = hog(c) = g(c) = d$. Therefore, $N$ has fixed point property, hence fixed point is a topological property.

*The solution of the problem is given by two redders and the details is as follow: 1) Mr. Sai Sameer M Mahadikar, Student PG Dept of Mathematics,Basaveshwar Science college, Bagalkot, Email: saiseanpaul@gmail.com 2) Dr. Bassayya Mathad, Assistant Professor, Dept. of Mathematics, S.G. Balekundri Institute of Technology, Belagavi. Email:bbmath.mathad@gmail.com*

## Down The Memory lane

This is about our $20^{th}$ anniversary celebrations at MSI Belagavi. In the Down the memory lane column we take the readers through some memorable old event highlighting its importance. Some available pictures also are highlighted. In the year 2012, the then PM Dr. Manmohan Singh declared that every $22^{nd}$ December i.e Ramanujan's birth anniversary, it will be called National Mathematics Day. MSI Belagavi used this occasion to celebrate and bring people together. During October that year we organised in assocciation with the school of mathematics and computing a National level Conference on Quantitative Finance.

The principal speakers were Prof. Anindia Goswami, IISER Pune who has worked extensively on stochastic control theory and applications to Finance, Rajendra Belgaumkar, industrialist and educationist who had a long stint as Risk management expert, Professor Raju's talk was about elementary discrete time processes, which can be used to understand how asset prices change over time. He also created a detailed dictionary of finance and economics terms to help mathematics researchers who are interested in studying finance.

In December 2012, the main event around S. Ramanujan's birth anniversary, Professor Srikrishna G Dani was invited. S. G. Dani who has been actively involved with NBHM, the National Board of higher mathematics and several other committees associated to IMU addressed the gathering. Incidentally Prof. Dani hails from Belagavi region and he did his schooling and secondary education at this place.

In his technical talk he spoke on Diophantine approximations. The other speaker was Prof. Bhargava Srinivasamurthy from the University of Mysore (he is now retired). Professor Bhargava who has led and contributed to the legacy of Srinivasa Ramanujan spoke about complex analytic facts arising in Number theory. He briefly covered the millenium prize problem viz Riemann Hypothesis in his talk. Prof. Bhargava has worked in the fields of Analysis, Number Theory, and Applied Mathematics. He mainly worked in the specific field of analytic number theory in association with Bruce Brendt. Following Bhargava several of his colleagues and students went on to explore the so called lost notebooks of Srinivasa Ramanujan.

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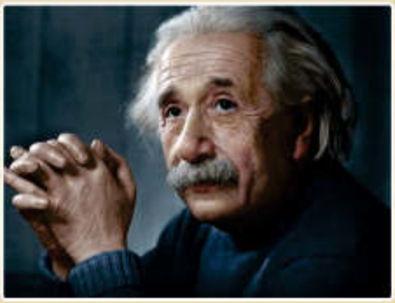
**(12 - 12-1939 to 24 - 03 - 2023)**



I maintain that cosmic religious feeling is the strongest and noblest incitement to scientific research. Only those who realize the immense efforts and, above all, the devotion which pioneer in theoretical science demands, can grasp the strength of the emotion out of which alone such work, remote as it is from the immediate realities of life, can issue. What deep conviction of the rationality of the universe and what a yearning to understand, were it but a feeble reflection of the mind revealed in this world, Kepler and Newton must have to enable them to spend years of solitary labor in disentangling the principles of celestial mechanics!

But the scientist is possessed by the sense of universal causation. The future, to him, is every whit as necessary and determined as the past. There is nothing divine about morality, it is a purely human affair. His religious feeling takes the form of a rapturous amazement at the harmony of natural law, which reveals an intelligence of such superiority that, compared with it, all the systematic thinking and acting of human beings is an utterly insignificant reflection.

However much our political convictions may differ, I know that we agree on one point: in the progressive achievements of the European mind both of us see and love our highest good. Those achievements are based on the freedom of thought and of teaching, on the principle that the desire for truth must take precedence of all other desires.

**- Some thoughts by Albert Einstein**

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